Electrodynamic Measurements, Especially on the Energy of Interaction

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Abstract

English translation of Wilhelm Weber's 1878 paper "Elektrodynamische Maassbestimmungen insbesondere über die Energie der Wechselwirkung", [Web78]. This work is the seventh of Weber's eight major Memoirs on "Elektrodynamische Maassbestimmungen", *Electrodynamic Measurements*.

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By Wilhelm Weber[1,2,3]

Of the general fundamental law of electric interaction announced in the *Electrodynamic Measurements* in the year 1846,[4,5,6] to which in Poggendorff's *Annalen* 1848, Vol. 73, p. 299,[7,8] was added the *potential* of the electric force derived from it, Helmholtz had claimed,[9] and William Thomson, Tait and others had agreed to,[10] that it is in contradiction to the principle of the conservation of energy; but C. Neumann and Maxwell did the opposite[11] by pointing out the error done by Helmholtz when he made the statement that the principle of the conservation of energy is only valid for forces that depend *solely* on the distance.[12,13]

Helmholtz has then established a completely *new* principle of energy,[14] whose difference to the *ordinary* principle of energy was specified by Neumann with the following words:[15]

> "While the *ordinary* principle of energy requires from any material system the existence of an *energy function*, i.e. the existence

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[1] [Web78].

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[3] The Notes by Wilhelm Eduard Weber are represented by [Note by WEW:]; the Notes by H. Weber, the Editor of Volume 4 of Weber's *Werke*, are represented by [Note by HW:]; while the Notes by A. K. T. Assis are represented by [Note by AKTA:].

[4] [Note by WEW:] See *Abhandlungen bei der Begründung der Königl. Sächs. Gesellschaft der Wissenschaften*. Leipzig 1846.

[5] [Note by HW:] Wilhelm Weber's *Werke*, Vol. III, p. 25.

[6] [Note by AKTA:] [Web46] with partial French translation in [Web87] and a complete English translation in [Web07].

[7] [Note by HW:] Wilhelm Weber's *Werke*, Vol. III, p. 245.

[8] [Note by AKTA:] [Web48] with English translation in [Web52], [Web66] and [Web19].

[9] [Note by AKTA:] [Hel47] with English translation in [Hel66].

[10] [Note by AKTA:] [TT67].

[11] [Note by AKTA:] [Neu68], [Max73, Chapter XXIII, Articles 852-853, pp. 429-430] and [Max54, Chapter XXIII, Articles 852-853, pp. 483-484].

We discussed Maxwell's points of view related to Weber's electrodynamics in [Ass94, Section 3.6, pp. 73-77].

[12] [Note by WEW:] See also Ad. Mayer: "Ueber den allgemeinsten Ausdruck der inneren Potentialkräfte eines Systems bewegter materieller Punkte, welches sich aus dem Princip der Gleichheit von Wirkung und Gegenwirkung ergiebt". [Translation: "On the most general expression of the internal potential forces of a system of moving material points, which results from the principle of the equality of action and reaction".] *Mathematische Annalen*, Vol. 13, p. 20.

[13] [Note by AKTA:] [May78].

[14] [Note by AKTA:] [Hel73].

[15] [Note by AKTA:] [Neu75, pp. 216-217] and [Neu77, p. 322].

> of a function depending on the momentarily state of the system, a function which has the property to increase in any time interval by exactly the quantity of work fed into during this interval, — the *new* principle established by Helmholtz requires not only the *existence* of such a function, but simultaneously a certain special *behavior* of that function, by claiming that the *'kinetic part of this function (the part that depends on velocity) must be always positive'*."

With respect to this Neumann also comments:

> "There is no doubt that it lies in the nature of the principles of physics that these are dilatable and flexible. The *principle of vis viva*[16] has slowly extended to the *principle of energy* and is possibly even further expandable."

In fact, it lies wholly in the essence and progress of *experimental research*, to already utilize such a principle as a *guideline* even when the *ultimate* formulation is still missing and can only later be extracted from the results of research; but since it is clear that the principle, in order to serve as a guideline for research, must nevertheless be *formulated*, which therefore could only be tentative, it results certainly that *during* this research such principle is really dilatable and flexible.

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[16] [Note by AKTA:] From Latin, meaning *living force* (*lebendige Kraft* in German). Term coined by G. W. Leibniz (1646-1716). It is an uncountable noun and therefore usually takes no plural form, although sometimes its plural is written as vires vivae.

Originally the *vis viva* of a body of mass m moving with velocity v relative to an inertial frame of reference was defined as mv^2, that is, twice the modern kinetic energy.

What Weber calls the living force (*lebendige Kraft*) of a particle should be understood as the modern kinetic energy, namely, $mv^2/2$. For instance, in his paper of 1871 on the conservation of energy he discussed two electrified particles of charges e and e' separated by a variable distance r. He then said the following, [Web71, Footnote 1, pp. 256-257 of Weber's *Werke*] with English translation in [Web72, p. 9]:

> If ε and ε' denote the masses of the particles e and e', and α and β the velocities of ε in the direction of r and at right angles thereto, and α' and β' the same velocities for ε', so that $\alpha - \alpha' = dr/dt$ is the relative velocity of the two particles, then
>
> $$\frac{1}{2}\varepsilon\left(\alpha^2+\beta^2\right)+\frac{1}{2}\varepsilon'\left(\alpha'^2+\beta'^2\right)$$
>
> is the total *lebendige Kraft* [expression translated as *vis viva* in [Web72, p. 9]] of the two particles.

The same reasoning will be expressed in this paper on Footnote 33 on page 17.

If by this Helmholtz was entitled to formulate the principle of energy tentatively in such a way that my fundamental law, condemned by him, gets in contradiction with it, then obviously the opposite is equally legitimate, namely, to formulate the same principle *tentatively* in such a way that it is not only *in agreement* with that fundamental law, but that the latter even results as a necessary consequence of the principle by proving that all *electrodynamic* laws, to which that fundamental law belongs, can be derived via the *tentative* principle from the *electrostatic* laws. To try this is the intention of the present treatise whereby, instead of the general fundamental law of the *electric interaction* established in the first treatise which simultaneously embraced electrostatics and electrodynamics, the principle of the conservation of energy will be put in the first place, from which then, in connection with the *static* fundamental law of the interaction of two particles, it shall be deduced, *firstly*, that fundamental law of the electric interaction and, *secondly*, the existence of an *energy function* for every pair of particles from which follows the validity of the *ordinary* energy principle, as it was spelled out by Neumann.

1 Guideline of the Experimental Research in Electrodynamics

After having obtained the *general laws of motion of bodies* as a base, in physics it essentially only remained to explore the *laws of the interaction of bodies*; because without interaction all bodies would *remain forever* in the state of rest or motion, in which they are. All changes of these states and all resulting phenomena are therefore *consequences* of their interactions.

Such interactions take place when the bodies *touch* each other, and also when they are *at a distance* from one another, and it was obvious that one had to start at investigating the latter, in order to extract a *guideline* for the former, which becomes rather necessary whenever the spacial situation of the bodies is not directly observable, as happens in the case of interactions of bodies touching one another. Indeed, it actually happened this way, one began with the investigation of the interactions of the *celestial bodies*, that is with the *gravitational interactions*.

After this first area of successful investigation of the *interaction of bodies*, namely *gravitational interactions*, it followed the investigation of *electric and magnetic interactions*, because apart from the gravitational interactions these were the only ones which were performed by one body *at measurable distance* from another one and which themselves could be determined by measurement.

For a long time almost all theoretical investigations of *electricity* and *magnetism*, in particular those of Coulomb and Poisson,[17] used Newton's theory of gravitation as a guideline,[18] until finally as a consequence of Oersted's and Ampère's discoveries of the *equivalence of closed currents and magnets*,[19] a totally new guideline arose which, firstly, led to the *reduction of all magnetic interactions to electric interactions* and, secondly, led to the establishment of a *fundamental law of interaction of each two current elements*.

As a *third guideline* served the general idea of reducing the interaction of all bodies with each other to the sole *interaction of each two* of them, accordingly then also the interactions of *current elements* should be reducible to the sole interactions of *each two electric particles*. By experience this idea could in general, even refraining from that the opposite (namely, interactions of *three* or more bodies which would not be reducible to interactions of *each two*) would lead to infinite complications, be viewed in large circles well justified and confirmed.

The material particles to be taken into account by the interaction of two current elements were now essentially one positive and one negative electric particle in every current element, between which one could set apart four, among themselves independent, interactions of each two particles.[20] For the determination of these four interactions Coulomb-Poisson's fundamental law (emulated on the law of gravitation) offered itself which already stood the test in the whole area of electrostatics; but the accordingly determined four interactions do not result in an over-all interaction, instead all individual interactions cancel each other completely,[21] and therefore Ampère's fundamental law of action at a distance between current elements was not reducible to Coulomb-Poisson's fundamental law of interaction between each two electric particles.

But Coulomb-Poisson's fundamental law of interaction between each two electric particles had been established only for these two particles at *relative*

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[17][Note by AKTA:] Charles Augustin de Coulomb (1736-1806) and Siméon Denis Poisson (1781-1840). See [Cou85a], [Cou85b], [Cou85c], [Cou86], [Cou87], [Cou88], [Cou89], [Pot84]; [Poi12a], [Poi12b], [Poi13], [Poi25a], [Poi25b], [Poi22a] and [Poi22b].

[18][Note by AKTA:] [New34], [New52] and [New99].

[19][Note by AKTA:] See [Oer20], [Oer65] and [Fra81]; [Amp23] and [Amp26] with a complete and commented English translation in [AC15].

[20][Note by AKTA:] That is, the positive particle of the current element 1 interacting with the positive and negative particles of the current element 2, together with the negative particle of the current element 1 interacting with the positive and negative particles of the current element 2.

[21][Note by AKTA:] Due to the charge neutrality of both current elements the sum of these four Coulombian interactions add up to zero.

rest or, at least, could only be justified as being in agreement with experience for such particles. In contrast, the four electric particles in two current elements form four pairs of particles which are not in relative rest, but in *relative motion*. Therefore this pointed to the conjecture that Coulomb-Poisson's fundamental law of interaction between any two electric particles, *if these particles are in relative motion*, still requires a *correction* which shall be denoted by x. Denoting the corrections of the above four interactions in the order x_1, x_2, x_3, x_4, then the sum of them should be non-zero and equal to the force determined by Ampère's law.

In this way it has now been found that — denoting two arbitrary electric particles in absolute measure by e and e',[22] and their relative distance, velocity, and acceleration by r, dr/dt, and d^2r/dt^2, and distinguishing these four values for the four pairs considered in two current elements through the indices 1, 2, 3, 4 — the repulsive force of two current elements determined by Ampère's law, namely

$$\frac{\alpha\alpha' ii'}{r^2}(3\cos\vartheta\cos\vartheta' - 2\cos\varepsilon) \ ,$$

(where α and α' are the lengths, i and i' the current intensities of the two current elements, r their distance, ϑ and ϑ' the angles which connect α and α' with r, and ε the angle formed by α and α'), is really represented through the sum

$$x_1 + x_2 + x_3 + x_4 \ ,$$

if one sets

$$x = \frac{1}{c^2} \cdot \frac{ee'}{r^2}\left(2r\frac{d^2r}{dt^2} - \frac{dr^2}{dt^2}\right) \ .$$

Here c denotes a *constant*, namely that relative velocity of two electric particles, at which, while it remains unchanged, no interaction takes place.[23]

In order to prove this, it is only necessary to express the quantities α, α', i, i' and the angles ϑ, ϑ' and ε related to the current elements as functions of the quantities e, e', r, dr/dt, d^2r/dt^2 with respect to the four pairs of particles.

This *correction* must hence be added to the repulsive force determined by the fundamental law of Coulomb-Poisson, whenever it should be valid not

[22][Note by AKTA:] That is, e and e' are the values of the electric charges of the two particles expressed in the absolute system of units introduced by C. F. Gauss (1777-1855) and Wilhelm Weber, [ARW04].

[23][Note by AKTA:] That is, if dr/dt remains constant and equals to c, then there will be no net force between the two electric particles interacting according to Weber's fundamental law.

only for pairs of particles at relative rest, but also for such motions which happen in current elements for which Ampère's law is valid.

But it is clear that those four electric particles can also be put in *manifold other relative motions*, other than those which take place in two current elements for which Ampère's law is valid. Indeed one can easily arrange setups in which two particles of positive and negative electricity are inside a current element and moving, instead of with *equal and constant velocity in opposite directions* (as Ampère assumed), either with equal but *variable* velocity in opposite directions, or with *unequal* velocities in directions which form an *arbitrary angle* with each other. All these different cases can easily be arranged, partly by letting the existing current in a conductor now to *disappear*, now to *arise* again, through opening or closing the circuit, partly by giving, to the oppositely moving electricities moving inside the conductor, a *joint motion with their conductor*.

If now the *corrected law* of Coulomb-Poisson is really valid in general for two electric particles, not just in relative rest or belonging to constant currents in resting conductors, but also for all their other motions, then therefrom it can be predicted and predetermined the action of current elements just as of individual particles — also in the just listed as well as in all other cases in which Ampère's law does not apply (which for a long time were left unnoticed and unobserved) — which serves as test and confirmation of the general validity of that law. That way really *all laws of Volta-induction* have been found,[24] in complete conformity with the phenomena observed by Faraday, and have been universally confirmed through manifold observation and measurements.

To this general fundamental law of interaction of two electric particles one can link further considerations about the *essence of the interaction*.

During all changes in the celestial bodies the *masses* of the bodies always remain unchanged, and also the *vis viva*'s of the bodies would, if there was no *interaction*, remain unchanged by the law of inertia. *Interactions* are therefore the reason of all changes of the vis viva, hence very obviously arises the question, if not vice versa the reason of all changes of interactions should be searched in the *vis viva*, so that amplification of the interaction can only be gained, if vis viva is lost, and that vice versa vis viva is only gained,

[24][Note by AKTA:] The expression utilized by Weber, *Volta-Induktion*, had been first suggested by Faraday himself in paragraph 26 of his first paper on electromagnetic induction published in 1831, see [Far52, §26, p. 267] and [Far11, p. 159]:

> For the purpose of avoiding periphrasis, I propose to call this action of the current from the voltaic battery, *volta-electric induction*.

This phenomenon of Volta-induction is nowadays called Faraday's law of induction.

if interaction suffers a decrease. *Interaction* of bodies would then be the *equivalent* of the lost vis viva, and *vis viva* the *equivalent* for lost interaction, whereby the *values* of interactions and vis viva would be brought in specific *dependence* of each other.

The general fundamental law of electric interaction mentioned above corresponds to this idea, because the dependence of the force resulting from the mutual interaction from the vis viva of the bodies is determined from it, in contrast to the Coulomb-Poisson law according to which there is no such dependence.

If one now calls the magnitude of the interaction of two particles their *energy of interaction*, and the magnitude of the relative vis viva of two particles their *energy of motion*,[25] there arises obviously the conjecture that at increase of one energy and simultaneous decrease of the other one, the gain of one energy grants compensation also *quantitatively* for the loss in the other energy, which presupposes the *homogeneity* of both energy quantities and means that *their sum is constant*. Denoting then by Q the relative vis viva of two particles and by P the energy of their interaction, one accordingly had to set

$$P + Q = a$$

where a is a constant for each *pair of particles*, just as mass is a constant for each *individual particle*.

It would thus be determined, *what in the interaction of two particles gets changed by mutual motion*, whereby it would be gained a foundation for the derivation of the *dynamic* law from the *static* one.

The total constant energy a would be at the same time the limit which could not be exceeded by the energy P, because indeed the energy Q (i.e. the vis viva of the particle) cannot have a value smaller than zero.

The conjecture pronounced here has suffered *several modifications*, consequently it is expressed differently in the tentatively given formulations of the *principle of the conservation of energy* which became the *guideline* of many researches in recent times, particularly in the theory of heat and elec-

[25] [Note by AKTA:] *Bewegungsenergie* in the original. This expression can be translated as "energy of motion" or "kinetic energy", see [Web71, p. 258 of Weber's *Werke*] with English translation in [Web72, p. 10].

tricity.[26,27] Given the importance and significance which this new *guideline* reached, some differences in the points of view and meanings deserve special

[26][Note by WEW:] It goes back to Thomas Young and W. Thomson to denote the sum of the *vis viva* and *heat* of a system of bodies together with the work determined by its *potential*, with the name of its *mechanical energy*, or shortly its *energy*, and this was then recognized and accepted by Clausius as being very practical.

Thomas Young, *Lectures on Natural Philosophy*, London 1807, Lecture VIII, says on page 78:

> "The term energy may be applied, with great propriety, to the product of the mass or weight of a body, into the square of the number expressing its velocity."

So Young denotes only the vis viva of a body (actually twice its value) with the name energy, but without explicitly adding that the body has only this one but no other energy. Rather this seems to suggest, since on the following page he uses for the vis viva of a body the more complete terminology 'energy of its motion', that a body may have, aside from its motion, another energy.

W. Thomson in *Phil. Magazine and Journal of Science*, IV. Series 9. London 1855, p. 523, says:

> "A body which is either emitting heat, or altering its dimensions against resisting forces, is doing work upon matter external to it. The mechanical effect of this work in one case is the excitation of thermal motions, and in the other the overcoming of resistances. The body must itself be altering in its circumstances, so as to contain a less store of work within it by an amount precisely equal to the aggregate value of the mechanical effects produced; and conversely, the aggregate value of the mechanical effects produced must depend solely on the initial and final states of the body, and is therefore the same whatever be the intermediate states through which the body passes, provided the *initial* and *final* states be the same. — The total mechanical energy of a body might be defined as the mechanical value of all the effect it would produce in heat emitted and in resistances overcome, if it were cooled to the utmost, and allowed to contract indefinitely or to expand indefinitely according as the forces between its particles are attractive or repulsive, when the thermal motions within it are all stopped."

Herein W. Thomson has enunciated with the *name energy* simultaneously the principle of the *conservation of energy*; because what a system of bodies looses from its stock of energy is gained by another system of bodies, from this obviously follows the *conservation of energy* in all systems of bodies taken together. — The same principle was essentially formulated earlier, only utilizing another expression, specifically by Helmholtz under the name of the *principle of conservation of force*.

[27][Note by AKTA:] [You07, Lecture 8, p. 78], [Tho53, p. 475 of the *Transactions of the Royal Society of Edinburgh*, p. 523 of the *Philosophical Magazine* and pp. 222-223 of the *Mathematical and Physical Papers*].

What Helmholtz called *principle of the conservation of force* [*Princip der Erhaltung der Kraft*], [Hel47] with English translation in [Hel66], is nowadays called *principle of the conservation of energy*.

attention.

The previous tentative formulation of the *principle of the conservation of energy* is fundamentally different, and it could easily appear to be in contradiction (which on closer inspection is not the case) with the formulation of the *"ordinary principle of energy"* of which C. Neumann says in Vol. XI, p. 320, of the *Mathematische Annalen*:[28]

> "This principle requires that for every material system there exists an *energy function*, i.e. a function depending on the momentarily state of the system, which has the property to increase in any given time interval by precisely the amount of work that is added to the system from the outside. At the same time we notice that this *energy function* (which one simply calls the *energy* of the system), based on Weber's law [this is Neumann's terminology for the above described corrected Coulomb-Poisson law], is represented by the sum of vis viva and potential ... Helmholtz, in the mean time, takes a somewhat different position for this question ... namely, in the article (*Monatsber. d. Berl. Akad.*, 18. April 1872) it says:
>
> > "One has usually been content in the investigations about whether the law of the conservation of energy is valid or not in certain processes of nature, to investigate if, when I may express the analytical result in words of practice, an infinitely repeated cyclic process can generate or destroy work. — Now in this sense Weber's hypothesis does not violate the law of the conservation of energy; but it does so in another sense — — —"

Neumann continues:

> "The following objection no longer concerns the ordinary energy principle, but a completely *new* one, a principle formulated here for the first time. Namely, while the ordinary energy principle requires for each material system the existence of an energy function, i.e. the existence of a function which has the property to

[28][Note by AKTA:] [Neu75, pp. 214-217] and [Neu77, pp. 320-322]. When Weber mentioned from where the quotation of Helmholtz came from, he quoted [Hel72a] with English translation in [Hel72b], but the correct reference is: [Hel73, p. 36 of the *Journal für die reine und angewandte Mathematik* and p. 649 of the *Wissenschaftliche Abhandlungen*].

> increase in any time interval by precisely the amount which equals the work added to the system in the interval, — that new principle not only requires the *existence* of such a function, but simultaneously certain special *qualities* of it by claiming that the *kinetic part* of this function (the part which depends on velocity) must always be *positive*."

Neumann adds in a note the following remark, already cited above:[29]

> "There is no doubt that the physical principles are incapable of a fixed formulation, that in their nature these are dilatable and flexible. The *principle of vis viva* has slowly extended to the *principle of energy* and is possibly even further expandable. — Accordingly it is a priori not impossible that this *energy principle* gradually extends to that *new principle* of Helmholtz. It only appears useful to me, at least temporarily, to denote both principles with different names." —

That last remark applies not only to Helmholtz's principle, but also to the one established above which also deviates from the *ordinary* one, whence for its better distinction so far it was already used the name principle of the *conservation* of energy, because according to it the *whole energy*, namely that the sum of the motion and the interaction, is really *conserved* unchanged, while according to the ordinary energy principle there exists only an *energy function*, whose magnitude is not at all conserved unchanged, but has the property to increase in any time interval by precisely the amount of work added to the system from the outside. Only in two special cases the ordinary principle can be considered, too, as a principle of the *conservation* of energy, namely in the case where the considered system contains *all* bodies in the world, and also in the case where the considered system is to be viewed as *completely isolated*, the reason is that in these two cases there are no *external* influences.

But given that diversity, it must be proved if there is no *contradiction* between the principle of the *conservation* of energy and the *ordinary* energy principle, as defined by Neumann, for which, as is easily seen, it is only needed to be shown that the energy of interaction P increases in any time interval by precisely the difference, in that time interval, of the increase of the potential V and the work S added to the pair of particles from the outside, i.e. that $dP = dV - dS$, which by taking into account the equation given

[29] [Note by AKTA:] [Neu75, pp. 216-217] and [Neu77, p. 322].

by the principle of the conservation of energy, namely $P + Q = a$, where a denotes a constant, leads to the *ordinary* energy principle, namely

$$d(Q + V) = dS \ ,$$

where $(Q + V)$ denotes Neumann's *energy function.* — One will attempt to provide this proof in Section 4 below.

The objective which shall be reached with this new, different from both the *ordinary* and the energy principle formulated by Helmholtz, consists essentially in

> *obtaining a principle which determines what gets changed in the interaction of bodies through their motion.*

Interaction only takes place between two bodies and *only* suffers a change through the relative vis viva of their motion. Presupposing this, and in addition that this *interaction* of two bodies or body particles is a quantity *homogeneous* with its relative vis viva, which forms with the magnitude of this vis viva the constant *energy sum* a, then a obviously means the magnitude of the interaction of the both particles *at rest*, i.e. their *static* interaction, and the principle of the conservation of energy is then the law through which it is determined that this *static* interaction decreases by Q, as a consequence of relative *motion* given by any vis viva of magnitude Q.

The *general fundamental law of electric interaction* would be immediately *as such* completely replaced by the principle of the conservation of energy and turned into a theorem, which would be derived and proved by means of the *principle of the conservation of energy.*

2 Interaction Energy Reduced to Absolute Measure

It is clear that from the equation established in the previous Section, in which tentatively the principle of the conservation of energy was formulated, namely

$$P + Q = a \ ,$$

the *energy of motion* Q can be determined, if the *energy of interaction* P is given, and vice versa; at the same time it is clear that the meaning of the equation, as the formulation of a principle, rests on the *physical meaning*, which is to be related to the notion of every *individual* energy, from which it must be clear the possibility of the determination of the magnitude of every energy independent of the others. For the *energy of motion* such a determination has long since been given; it is required therefore only a similar determination for the *interaction energy*.

The interaction of two particles during a distance change consists in *work*. Without distance change interaction takes place, but not *work*; yet the pair of particles always possesses a *capability to work*,[30] i.e. the property, to perform work through distance changes. In that *capability to work* one recognizes the interaction and its magnitude gives the scale for the *energy* of the interaction.

Determination of the capability to work must be built on *work measurement*. But now *work* consists either in *cancellation* of opposite work, or in *generation* (or annihilation) of *vis viva*. Works, which cancel each other, elude direct measurement; in contrast increase or decrease of vis viva is under suitable conditions subject of direct observation and measurement, to which ultimately all *work measurement* is reduced to.

If accordingly *work* is determinable from the measurable vis viva generated by it, in case it is not neutralized by an opposite work, then for the *work capability* of a pair of particles it suffices to determine the work magnitude which would be achieved by the particle *interaction* during a certain, still to be determined, change of distance. If this work magnitude happens to be positive or negative is not taken into consideration, therefore the *absolute value*[31] of this work magnitude serves as a *measure* of the work capability.

In contrast, according to the established principle of the conservation of energy, for work capability one has to take into consideration the velocity dr/dt with which the change of distance takes place. The reason is that it is with this velocity that the energy of motion changes, consequently according to the mentioned principle also the interaction energy P. From this it is clear that the energy P, i.e the work capability of a pair of particles, is only exactly measurable for a *given value of the velocity* dr/dt, and that this value has to be assumed *constant* during the corresponding change of distance.

But because at a constant value of dr/dt there does not happen neither increase nor decrease of vis viva, usable for *direct* work measurement, one has to look for an *indirect* method to determine the work capability. If during a change of distance one wishes not to cause a change of relative velocity through interaction, then the work performed during the change of distance via *interaction* must be neutralized by the one performed through *external*

[30] [Note by AKTA:] *Arbeitsvermögen* in the original.

[31] [Note by WEW:] From this it follows that the validity of the principle $P + Q = a$ is limited to the cases in which Q does not exceed the value of a. But all vis viva Q of the known bodies are however such small fractions of a, that most likely the case $Q > a$ does not appear. According to our present knowledge it would be necessary [for this case to take place] two bodies with a relative velocity $> 439\,450$ kilometer/[second].

influence, and the latter one can be used — if it is of known origin and so it is exactly determined, e.g. if it arises from known weights which act on the particles during the change of distance — to *indirectly* measure the work performed by the interaction.

The work performed via interaction during the change of distance dr of two particles e and e' is given in absolute value by $\pm[\partial V/\partial r]dr$,[32] where V denotes the *potential* of the pair of particles and the upper or lower sign is valid depending if the product ee' is positive or negative. Similarly the work performed during a distance change from ρ' to ρ'' is given by

$$\pm \int_{\rho'}^{\rho''} \frac{\partial V}{\partial r} dr \ .$$

The task to reduce to absolute measure the energy of interaction of two particles e and e', i.e. the determination of their work capability by absolute measure, is thereby reduced to finding the integral value $\pm \int_{\rho'}^{\rho''} \frac{\partial V}{\partial r} dr$ in which only the integration limits ρ' and ρ'' need to be determined.

As by *work capability* one understands the value of the work performed via interaction during an *exactly to be determined distance change*, it is then clear that the distance borders ρ' and ρ'' in the expression $\pm \int_{\rho'}^{\rho''} \frac{\partial V}{\partial r} dr$ must get *exactly determined and constant values*, from which it follows that these borders cannot be the same as those of the potential $\int_{\infty}^{r} \frac{dV}{dr} dr$ of which one, namely r, is variable.

Since further this is about determining the *whole work capability*, associated to the pair of particles via the interaction of its particles, it is clear that the borders should be placed apart as much as possible, as far as possible without getting in contradiction with the principle of the conservation of energy, according to which the *energy sum* of the pair of particles should be a constant a, and this constant should also be the limit which the *energy of interaction* must not exceed and should only be reached when the *energy of motion* is zero.

From here results first the determination of one border $\rho' = \infty$; concerning the other border ρ'', its value must not be smaller than the one for which it would be valid $\pm \int_{\infty}^{\rho''} \frac{\partial V}{\partial r} dr = a$. Let now ρ denote the resulting value of ρ''.

In order to determine the *energy P of interaction* of two particles e and e' whose masses were denoted by ε and ε' and whose energy of motion is $Q = \frac{1}{2}[\varepsilon\varepsilon'/(\varepsilon + \varepsilon')] \cdot [dr^2/dt^2]$, one then gets the equation

$$P = \pm \int_{\infty}^{\rho} \frac{\partial V}{\partial r} dr \ .$$

[32] [Note by WEW:] The symbol of *partial derivation* has been chosen to indicate that in this differentiation dr/dt should be considered constant.

Here it is to be observed that, firstly, during the change of distance there should be, apart from the interaction, an *external* influence on the pair of particles which keeps *constant* in V the given value of $\partial r/\partial t$; secondly, that the upper or lower sign is valid depending if the product ee' is positive or negative; thirdly, that $P = a$ for $Q = 0$ which serves to determine ρ.

The formula for P can yet be transformed as follows. One can decompose $P = \pm \int_\infty^\rho \frac{\partial V}{\partial r} dr$ into two parts, namely:

$$P = \pm \int_\infty^r \frac{dV}{dr} dr \pm \int_r^\rho \frac{\partial V}{\partial r} dr \ ,$$

where the *first* part is the absolute value of the *potential* V, here we used the ordinary symbols of differentiation since it does not matter if dr/dt is treated as variable, or not. Concerning the *second* part one has to note that the *given* value of the velocity dr/dt must be presupposed constant during the change of distance from r to ρ.

Denoting by s the work performed by *external* influence during the change of distance from r to ρ, then in order to keep dr/dt constant, the following relation must necessarily be valid:

$$\pm \int_r^\rho \frac{\partial V}{\partial r} dr + s = 0 \ .$$

From this one gets the following formula to determine the energy P, namely

$$P = \pm V - s \ ,$$

where V denotes the *potential* of the particles of charges e and e' and s the work which must be performed during the change of distance from r to ρ by *external* influence, so that the *given* value of the relative velocity dr/dt remains unchanged.

3 Derivation of the Electrodynamic Potential Law from the Electrostatic One by Means of the Principle of Energy

According to the definition of both energies of a pair of electric particles e and e' whose masses were denoted by ε and ε' and their distance by r, namely[33,34,35]

$$\text{the } \textit{energy of motion } Q = \frac{1}{2}\frac{\varepsilon\varepsilon'}{\varepsilon+\varepsilon'}\cdot\frac{dr^2}{dt^2}\ ,$$

and

$$\text{the } \textit{energy of interaction } P = \pm\int_{\infty}^{\rho}\frac{\partial V}{\partial r}\,dr\ ,$$

— where the upper or lower sign is valid depending on the product ee' being positive or negative, and where with the given value of the magnitude Q the relative velocity dr/dt must be assumed *constant* during the change of distance and the same should be valid for Q, — there results from the principle of energy presented in Section 1, according to which $P + Q = a$ forms a *constant* sum, the following equation between the two constants a and ρ and the two variables Q and V, namely

$$\pm\int_{\infty}^{\rho}\frac{\partial V}{\partial r}\,dr = a - Q\ . \qquad (1)$$

[33] [Note by WEW:] If α and β denote the velocities of the mass ε in the direction r and the one orthogonal to it, while α' and β' denote the same velocities for ε', so that $\alpha - \alpha' = dr/dt$ is the relative velocity of the two particles, then

$$\frac{1}{2}\varepsilon\left(\alpha^2+\beta^2\right)+\frac{1}{2}\varepsilon'\left({\alpha'}^2+{\beta'}^2\right)$$

is the total vis viva belonging to the two particles. Set now for α,

$$\frac{\varepsilon\alpha+\varepsilon'\alpha'}{\varepsilon+\varepsilon'}+\frac{\varepsilon'(\alpha-\alpha')}{\varepsilon+\varepsilon'}\ ,$$

and for α',

$$\frac{\varepsilon\alpha+\varepsilon'\alpha'}{\varepsilon+\varepsilon'}-\frac{\varepsilon'(\alpha-\alpha')}{\varepsilon+\varepsilon'}\ ,$$

then one gets the total vis viva of the two particles as the sum of *two parts*,

$$=\frac{1}{2}\cdot\frac{\varepsilon\varepsilon'}{\varepsilon+\varepsilon'}\cdot\frac{dr^2}{dt^2}+\frac{1}{2}\left[\frac{(\varepsilon\alpha+\varepsilon'\alpha')^2}{\varepsilon+\varepsilon'}+\varepsilon\beta^2+\varepsilon'{\beta'}^2\right]\ ,$$

from which the *first*, namely $\frac{1}{2}[\varepsilon\varepsilon'/(\varepsilon+\varepsilon')]\cdot[dr^2/dt^2]$ is the *relative vis viva* of the two particles, which was denoted above by Q. — See *Abhandlungen der Königl. Sächs. Gesellschaft der Wissenschaften*, Vol. X, p. 12.

[34] [Note by HW:] Wilhelm Weber's *Werke*, Vol. IV, p. 257.

[35] [Note by AKTA:] [Web71, pp. 256-257 of Weber's *Werke*] with English translation in [Web72, p. 9].

Now according to the fundamental law of *electrostatics* the potential for $Q = 0$ is given by $V = ee'/r$. Inserting these values of the variables Q and V into Equation (1), then one obtains the following equation between the two constants a and ρ, reducing one to the other, namely

$$\pm \int_{\infty}^{\rho} \frac{d\frac{ee'}{r}}{dr}\, dr = a \ ,$$

from which one finds the value of the constant ρ, namely

$$\rho = \pm \frac{ee'}{a} \ . \qquad (2)$$

Inserting now this value of ρ into Equation (1), then there results the following equation between only *one* constant, namely a, the given value of the variable Q, and the value of the variable V which we are looking for, namely

$$\pm \int_{\infty}^{\pm\frac{ee'}{a}} \frac{\partial V}{\partial r}\, dr = a - Q \ , \qquad (3)$$

from which V is to be determined.

One easily observes that this Equation (3) is satisfied by V defined as follows

$$V = \frac{ee'}{r}\left(1 - \frac{Q}{a}\right) \ ;$$

indeed, substituting this values in the first term of Equation (3) and taking into consideration that, according to the definition given in Section 2, in the formula $P = \pm \int_{\infty}^{\rho} \frac{\partial V}{\partial r}\, dr$ the given value of the relative velocity dr/dt, hence also $Q = \frac{1}{2}[\varepsilon\varepsilon'/(\varepsilon + \varepsilon')] \cdot [dr^2/dt^2]$, must be assumed *constant* during the change of distance, then one finds for one limit, $r = \pm ee'/a$, the value $V = \pm a(1 - Q/a)$, and for the other limit $r = \infty$ the value $V = 0$, consequently the difference of these values

$$\int_{\infty}^{\pm\frac{ee'}{a}} \frac{\partial V}{\partial r}\, dr = \pm a\left(1 - \frac{Q}{a}\right) \ ,$$

therefore,

$$\pm \int_{\infty}^{\pm\frac{ee'}{a}} \frac{\partial V}{\partial r}\, dr = a\left(1 - \frac{Q}{a}\right) = a - Q \ ,$$

completely in agreement with Equation (3).

This formula of the *law of the electrodynamic potential* derived from the fundamental law of *electrostatics* with the help of the *principle of energy*, namely

$$V = \frac{ee'}{r}\left(1 - \frac{Q}{a}\right) \ , \qquad (4)$$

can yet be rewritten in the following way.

The *constant sum of energy* a is according to the principle of energy the limit of the kinetic energy $Q = \frac{1}{2}[\varepsilon\varepsilon'/(\varepsilon+\varepsilon')] \cdot [dr^2/dt^2]$ for decreasing values of the interaction energy P, i.e. one has $Q = a$ when $P = 0$. Denoting then by c the relative velocity dr/dt of the two particles for this *limit* of the kinetic energy a, there results

$$a = \frac{1}{2}\frac{\varepsilon\varepsilon'}{\varepsilon+\varepsilon'} \cdot c^2 \ .$$

Substituting now these values of $Q = \frac{1}{2}[\varepsilon\varepsilon'/(\varepsilon+\varepsilon')] \cdot [dr^2/dt^2]$ and $a = \frac{1}{2}\frac{\varepsilon\varepsilon'}{\varepsilon+\varepsilon'} \cdot c^2$ in Equation (4), one gets for the law of the electrodynamic potential the following expression:

$$V = \frac{ee'}{r}\left(1 - \frac{1}{c^2} \cdot \frac{dr^2}{dt^2}\right) \ .$$

Between the three *constants* a, ρ, c appearing in this derivation of the law of the electrodynamic potential of a pair of electric particles e and e', with masses ε and ε', the following relations finally take place, namely

$$a = \pm\frac{ee'}{\rho} = \frac{1}{2}\frac{\varepsilon\varepsilon'}{\varepsilon+\varepsilon'} \cdot c^2 \ .$$

For the *electrodynamic potential* V one gets by interchanging these constants the following formula:

$$V = \frac{ee'}{r}\left(1 - \frac{Q}{a}\right) = \frac{ee'}{r}\left(1 - \frac{1}{c^2} \cdot \frac{dr^2}{dt^2}\right) = \frac{ee'}{r} \mp \frac{\rho}{r} Q \ ,$$

where the upper or lower sign is valid depending if the product ee' is positive or negative.

4 Derivation of the Ordinary Principle of Energy from the Principle of the Conservation of Energy

The *ordinary* principle of energy, as it was formulated by Neumann, requires that for every material system there exists an *energy function*, i.e. a function depending only on the momentarily state of the system, which has the property to increase in each time interval by precisely the amount of work added to the system from the outside during that interval. This *energy function* one has often simply called the *energy*.

In case of a system of two particles at a distance r from each other, on which during the change of distance dr is exerted by mutual interaction the *internal* work Rdr, and by *external* influence the *external* work dS, the increase of the vis viva Q is according to a well known *general theorem of mechanics* precisely equal to the sum of all *internal* and *external* works exerted on the system, namely

$$dQ = Rdr + dS \ .$$

Hence if there is a function depending on the present state of the pair of particles and which has the property to increase during the change of distance dr by the amount $dQ - Rdr = dS$, then for such a pair of particles the *ordinary* principle of energy is valid.

Since now for a pair of electric particles e and e' it was proved using the *potential law* developed in the previous Section, under the assumption of the principle of the *conservation of energy*, that the *internal* work Rdr is the total differential of the function $-(ee'/r)(1 - dr^2/c^2dt^2)$, which depends just as Q only on the present state of the pair of particles, is is clear that the difference of the two quantities which also depends only on the present state of the pair of particles, namely

$$Q + \frac{ee'}{r}\left(1 - \frac{dr^2}{c^2dt^2}\right) \ ,$$

has the property to increase during the change of distance dr by $dQ - Rdr = dS$, whereby consequently for such a pair of particles not only the principle of the *conservation of energy* is valid, but also the *ordinary* principle of energy, and

$$Q + \frac{ee'}{r}\left(1 - \frac{dr^2}{c^2dt^2}\right) \ ,$$

is its *energy function*.

The simultaneous validity of both principles, namely the principle of the *conservation* of energy, stating that $P + Q = a$, and the *ordinary* principle of energy, stating that $d(Q + V) = dS$, where S denotes the work performed by *external* influence, presupposes, as already mentioned at the end of Section 1, firstly that

$$dP = dV - dS \ ,$$

or, since $P = \pm V - s$ according to Section 2, consequently one has

$$dP = \pm dV - ds \ ,$$

secondly that $\pm dV - ds = dV - dS$, which can easily be proved with the help of the equations obtained in Sections 2 and 3:

$$\pm \int_r^\rho \frac{\partial V}{\partial r} dr + s = 0 \ , \qquad (1)$$

$$V = \pm \frac{\rho}{r} (a - Q) \ , \qquad (2)$$

and with the help of the probative formulation of the *ordinary* principle of energy, already mentioned in Section 1, equation

$$dS = d(Q + V) \qquad (3)$$

can be easily proved. Namely, from (1) and (2) there result the equations

$$s = (a - Q) \left(\frac{\rho}{r} - 1 \right) \ ,$$

$$-ds = \rho(a - Q) \frac{dr}{r^2} + \left(\frac{\rho}{r} - 1 \right) dQ \ ,$$

$$\pm dV = -\rho(a - Q) \frac{dr}{r^2} - \frac{\rho}{r} dQ \ ,$$

from which follows

$$\pm dV - ds = -dQ \ .$$

But now $dV - dS = -dQ$ according to (3), consequently $\pm dV - ds = dV - dS$, which was to be proved.

5 The General Law of Electric Force

The *potential* of two electric particles e and e' at a distance r has been found in Section 3

$$V = \frac{ee'}{r} \left(1 - \frac{Q}{a} \right) \ ,$$

which one interprets as the *work* performed via interaction by the two particles of charges e and e' possessing relative vis viva Q, whenever they are displaced from infinite distance to distance r. The differential quotient dV/dr denotes then the *force* exerted by the two particles at a distance r through interaction, namely, an attracting or a repulsive force depending if this expression is positive or negative.

The relative vis viva Q of the two particles with masses ε and ε' is represented by

$$Q = \frac{1}{2}\frac{\varepsilon\varepsilon'}{\varepsilon + \varepsilon'} \cdot \frac{dr^2}{dt^2} ,$$

which indicates that Q is a function of time t (except when dr/dt is explicitly supposed to be constant), as well as r, and that consequently any of these two variables r and Q can also be considered as a function of the other one.

From this it results the *repulsive force*

$$-\frac{dV}{dr} = \frac{ee'}{r^2}\left(1 - \frac{Q}{a}\right) + \frac{ee'}{ar} \cdot \frac{\frac{dQ}{dt}}{\frac{dr}{dt}} ,$$

or, if one substitutes herein the values

$$Q = \frac{1}{2}\frac{\varepsilon\varepsilon'}{\varepsilon + \varepsilon'} \cdot \frac{dr^2}{dt^2} \quad \text{and} \quad a = \frac{1}{2}\frac{\varepsilon\varepsilon'}{\varepsilon + \varepsilon'} \cdot c^2 ,$$

from which follows

$$\frac{dQ}{dt} = \frac{\varepsilon\varepsilon'}{\varepsilon + \varepsilon'} \cdot \frac{dr}{dt} \cdot \frac{d^2r}{dt^2} = \frac{2a}{c^2} \cdot \frac{dr}{dt} \cdot \frac{d^2r}{dt^2} ,$$

it results the *repulsive force*

$$-\frac{dV}{dr} = \frac{ee'}{r^2}\left(1 - \frac{1}{c^2} \cdot \frac{dr^2}{dt^2} + \frac{2r}{c^2} \cdot \frac{d^2r}{dt^2}\right) .$$

But now the relative acceleration d^2r/dt^2 is composed of *two parts*, namely the part *depending* on the interaction of the two particles, and the therefrom *independent* part. Let the *latter* be denoted by f, the *former* multiplied by $\varepsilon\varepsilon'/(\varepsilon + \varepsilon')$ gives the repulsive force $-dV/dr$, and hence it can be represented by the quotient $-[(\varepsilon + \varepsilon')/\varepsilon\varepsilon'] \cdot [dV/dr]$. Therefore one has

$$\frac{d^2r}{dt^2} = f - \frac{\varepsilon + \varepsilon'}{\varepsilon\varepsilon'} \cdot \frac{dV}{dr} .$$

Substituting this value for d^2r/dt^2 in the above equation, and setting $\rho = \pm 2[(\varepsilon + \varepsilon')/\varepsilon\varepsilon'] \cdot [ee'/c^2]$ according to Section 3, where the upper or lower sign is valid depending if the product ee' is positive or negative, then one gets

$$-\frac{dV}{dr} = \frac{ee'}{r^2}\left(1 - \frac{1}{c^2} \cdot \frac{dr^2}{dt^2} + \frac{2r}{c^2} f\right) \mp \frac{\rho}{r} \cdot \frac{dV}{dr} ,$$

and from here finally the following expression for the *repulsive force*:

$$-\frac{dV}{dr} = \frac{ee'}{r(r \mp \rho)} \cdot \left(1 - \frac{1}{c^2} \cdot \frac{dr^2}{dt^2} + \frac{2r}{c^2} f\right) \ ,$$

where the upper or lower sign is valid depending if the product ee' is positive or negative. One can also write this expression in the form

$$-\frac{dV}{dr} = \frac{ee'}{r(r - \frac{ee'}{a})} \cdot \left(1 - \frac{1}{c^2} \cdot \frac{dr^2}{dt^2} + \frac{2r}{c^2} f\right) \ .$$

The expression of the electric force in this developed form can now serve for a better overview of the forces that each of the two particles exerts on the other one, but one has to bear in mind that in this form the forces cannot be composed according to the parallelogram law.[36,37] One recognizes e.g. immediately from here that for positive values of the product ee' this force would be infinitely large, not only for $r = 0$, but also for $r = \rho$; but also recognizes at the same time that in reality the last case $r = \rho$ never happens, because, no matter how large the relative velocity might be in any distance that differs from ρ by a finite value, yet $r = \rho$ will never take place.

Since ee' is positive, the force for $r > \rho$ is *repulsive* and increases, while r decreases and approaches the limit ρ, to *infinity*, from which it is clear that the approximation velocity, which although very large was not yet infinite, must be cancelled from this *repulsive force* increasing to infinity, already before one has $r = \rho$, and that then right away r starts increasing again. It follows from this, that in this case it never happens $r = \rho$, but that the two particles necessarily have to stay at a distance larger than ρ from one another.

For $r < \rho$ the force is *attractive* and grows, while r increases and approximates the limit ρ, *to infinity*, from which it is clear that the distant velocity which was, although very large not yet infinite, must be neutralized by the *attracting force* increasing to infinity, already before one has $r = \rho$,

[36][Note by WEW:] Since the components of the acceleration by forces, whose potential depends on the velocities of the moving points, are given by expressions which contain the accelerations themselves in such a form that the values of the latter can only be obtained by resolving the equations, one has after a remark of Carl Neumann to observe that while *before* the resolution one may compose the expressions of the accelerations in the presence of simultaneous action of several forces according to the common rules, that latter property gets lost after reformation of the expressions due to resolving the equations. Here accelerations getting infinite is characterized by the vanishing of the determinant formed from the coefficients of the accelerations in the individual equations. Cf. *Mathematische Annalen*, Vol. 11, p. 323, Note.

[37][Note by AKTA:] [Neu77, Note on p. 323].

and that then right away r starts decreasing again. Hence in this case the two particles always will stay at a distance smaller than ρ from one another.

If a similar restriction of motion, like the one for two particles which meant that they had to stay at a distance less that ρ once they were that close, should be encountered also for a larger number of particles in a small region, which would mean that all these particles had to stay together in that same small region, then such a larger number of particles would form together, just as those two, a *molecule* and in the same way and under appropriate circumstances also the particles outside this molecule could be joined together into *molecules*. All these molecules, as it is clear, must be separated of one another by gaps of at least the size ρ, and they would repel one another. But it would need further investigations to decide if, and under which circumstances, a *system of such molecules* could rest in a stable equilibrium, and, if this should be the case, according to which laws small perturbations of the equilibrium would propagate, in order to decide the question, if the *luminiferous ether* and the *light waves* in space could not be based on and explained by a stable aggregate state of such *molecules* distributed in the celestial space and composed of electric particles.

It is common to call that force which is exerted by two electric particles e and e' at distance r from one another, when they are at *relative rest*, as their *electrostatic force* and determine it according to the *electrostatic law*, namely $= ee'/r^2$. But two particles are at *relative rest*, if their relative velocity $dr/dt = 0$ vanishes. Now it results from the obtained *general law* of the repulsive force of two electric particles that the magnitude of the force is given not by $= ee'/r^2$, but by

$$= \frac{ee'}{r(r \mp \rho)} \cdot \left(1 + \frac{2r}{c^2} f\right) ,$$

where f is that part of its relative acceleration which is *independent* of the *interaction* of the two particles, i.e. the sum of that acceleration $= \alpha^2/r$ which arises, firstly, from the relative velocity α of the particles in some direction orthogonal to r and, secondly, from that acceleration $= [(\varepsilon + \varepsilon')/\varepsilon\varepsilon']\, \Delta$ which arises from the difference Δ of the *external* forces decomposed according to r and exerted on the two particles e and e', where ε and ε' are the masses of the two particles.

But even in the case that both particles are not only at relative rest, but also the part f of their acceleration that is *independent* of their interaction is $= 0$, their repulsive force still turns out different from the value ee'/r^2 determined by the *electrostatic law*; according to the above *general law* the repulsive force is $= [ee'/r(r \mp \rho)]$.

In order to get according to this *general law* the value ee'/r^2 determined by the *static law*, the part of the acceleration denoted by f must not be $= 0$, but must be opposite to the other part that depends on the interaction of the two particles, namely equal to $[(\varepsilon + \varepsilon')/\varepsilon\varepsilon'] \cdot [ee'/r^2]$, i.e.

$$f = -\frac{\varepsilon + \varepsilon'}{\varepsilon\varepsilon'} \cdot \frac{ee'}{r^2} = \mp \frac{\rho c^2}{2r^2} \, .$$

With this value of f one finds according to the *general law*, in the case $dr/dt = 0$, the magnitude of the repulsive force:

$$-\frac{dV}{dr} = \frac{ee'}{r(r \mp \rho)} \cdot \left(1 + \frac{2r}{c^2} f\right) = \frac{ee'}{r(r \mp \rho)} \cdot \left(1 \mp \frac{\rho}{r}\right) = \frac{ee'}{r^2} \, ,$$

that is, equal to the value determined by the *electrostatic law*. Therefore a real static equilibrium between two particles at relative rest only happens, if the acceleration resulting from their interaction gets neutralized by the acceleration that is independent of their interaction.

6 Laws of Motion of Two Electric Particles Impelled Only by Their Action on Each Other

The laws of motion of two electric particles impelled only by their action on each other, which were already developed in the *Electrodynamic Measurements*, Vol. X of these *Abhandlungen*,[38,39] shall here only be considered in detail and represented graphically in the case in which these particles have no relative motion orthogonal to their connecting segment, and this serves to refute erroneous conclusions drawn from the fundamental law.

According to Section 3 the *general potential* of two electric particles e and e' at a distance r from one another was

$$V = \frac{ee'}{r}\left(1 - \frac{1}{c^2} \cdot \frac{dr^2}{dt^2}\right) \, ,$$

where the *repulsive force* of the two particles, as mentioned in the previous Section, has been represented by $-dV/dr$; but if this repulsive force is to be represented by $+dV/dr$, then the potential

$$V = \frac{ee'}{r}\left(\frac{dr^2}{c^2\, dt^2} - 1\right)$$

[38] [Note by HW:] Wilhelm Weber's *Werke*, Vol. IV, p. 268.

[39] [Note by AKTA:] [Web71, Section 8, p. 268 of Weber's *Werke*] with English translation in [Web72, Section 8, p. 119].

has to be set.

According to the latter the acceleration of the particle e in the direction r is given by $= [1/\varepsilon] \cdot [dV/dr]$, and the acceleration of the particle e' in the opposite direction by $= [1/\varepsilon'] \cdot [dV/dr]$, where ε and ε' are the masses of the particles e and e', from where results the *relative acceleration* of the two particles,

$$\frac{d^2r}{dt^2} = \left(\frac{1}{\varepsilon} + \frac{1}{\varepsilon'}\right) \cdot \frac{dV}{dr} \ .$$

From this results by multiplication with $2dr$ the differential equation

$$2\frac{dr}{dt} \cdot \frac{d^2r}{dt^2} = 2\left(\frac{1}{\varepsilon} + \frac{1}{\varepsilon'}\right) \cdot \frac{dV}{dr}\, dr \ ,$$

and via integration from $r = r_0$ to $r = r$, where r_0 denotes the value of r for which the relative velocity $dr/dt = 0$, we get

$$\frac{d^2r}{dt^2} = 2\left(\frac{1}{\varepsilon} + \frac{1}{\varepsilon'}\right) \cdot \left[\frac{ee'}{r}\left(\frac{dr^2}{c^2\, dt^2} - 1\right) + \frac{ee'}{r_0}\right] \ ,$$

or, if we set $dr/c\,dt = u$ and $2([1/\varepsilon] + [1/\varepsilon']) = \pm\rho c^2/ee'$,

$$u^2 = \pm\rho\left(\frac{1}{r}\left(u^2 - 1\right) + \frac{1}{r_0}\right) \ ,$$

where the upper or lower sign is valid depending if the product ee' is positive or negative.

Considering now the case where ee' is positive, and expressing the distances r and r_0 in terms of the constant ρ associated above to the pair of particles, one gets

$$u^2 = \frac{1}{r}(u^2 - 1) + \frac{1}{r_0} \ .$$

If for such a pair of particles the distance r and the velocity u are determined at whatever time, then from the equation above it results

$$r_0 = \frac{r}{1 - (1 - r)u^2} \ .$$

If in this way it has been determined for the considered pair of particles, which moves only through their mutual interaction, that distance r_0 for which $u = 0$, then from the equation

$$u^2 = \left(1 - \frac{r}{r_0}\right)\frac{1}{1 - r} \ ,$$

one can find all values of r and u for the given value of r_0 if one inserts for r any arbitrary series of increasing values from $r = 0$ to $r = \infty$.

Such a series of related values of r and u is graphically represented by a curve, whose abscissa and ordinate represent the related values of r and u.

But the value of r_0 for the same pair of particles can be very different at different times, if in the intermediate time apart from their mutual interaction also external influences occurred. For every other value of r_0 it results, after eliminating any external influence, another series of related values of r and u, which are represented graphically by another curve.

In accordance with this one gets for a whole system of different values of r_0 the following Table of related values of r and u, ordered according to these values, and a corresponding system of curves, shown in Figure 1.[40]

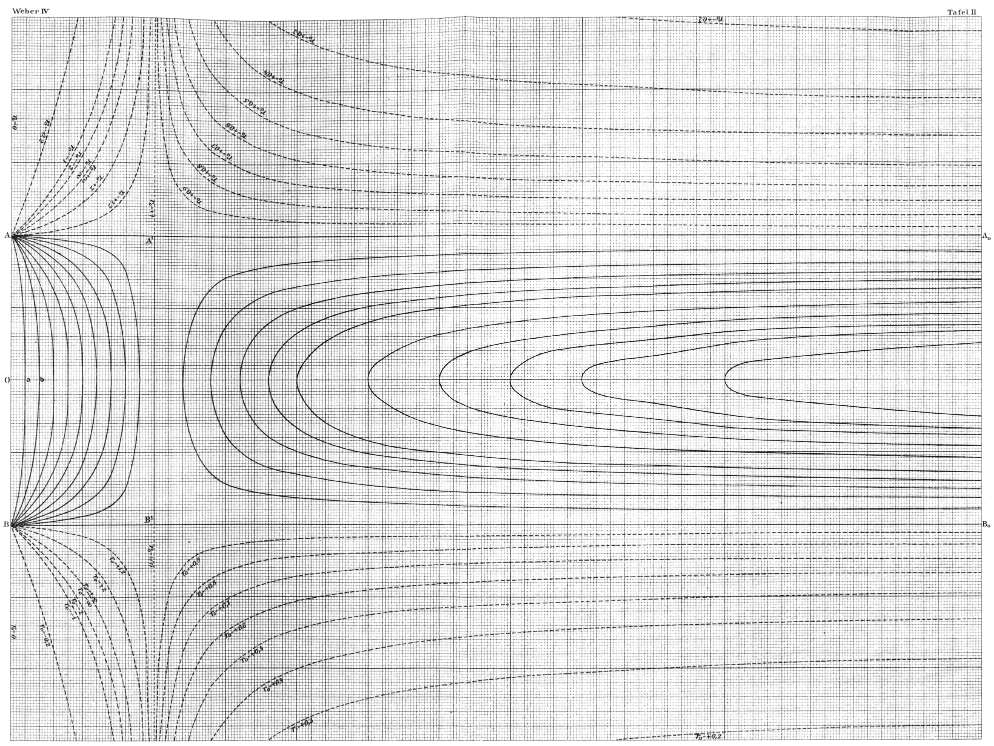


Figure 1: Ordinate $\pm u$ as function of abscissa r.

Here it is to be remarked that in this graphic representation the values of r in the following Table are shown as abscissas associated to ordinates $\pm u$, more precisely those of $+u$ as *positive* and those of $-u$ as *negative*, in order

[40][Note by AKTA:] A larger image appears on page 55.

to distinguish the *distance* of the particles from their *approach.* The *first* section of the Table corresponding system of curves fills the space $AA'B'B$, the one corresponding to the *second* [Section fills] the space $A'A_0B_0B'$, which has to be extended to infinity on the side of A_0B_0.

Values of $u = \sqrt{\frac{1}{1-r}\left(1 - \frac{r}{r_0}\right)}$ for values of r and r_0 between 0 and 1:

$r_0 =$	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
$r = 0.0$	0.00	±1.00	±1.00	±1.00	±1.00	±1.00	±1.00	±1.00	±1.00	±1.00	±1.00
$r = 0.1$		0.00	±0.75	±0.86	±0.91	±0.95	±0.97	±0.98	±0.99	±0.99	±1.00
$r = 0.2$			0.00	±0.65	±0.79	±0.86	±0.91	±0.94	±0.97	±0.98	±1.00
$r = 0.3$				0.00	±0.60	±0.75	±0.84	±0.90	±0.94	±0.97	±1.00
$r = 0.4$					0.00	±0.57	±0.75	±0.84	±0.91	±0.96	±1.00
$r = 0.5$						0.00	±0.57	±0.75	±0.86	±0.94	±1.00
$r = 0.6$							0.00	±0.60	±0.79	±0.91	±1.00
$r = 0.7$								0.00	±0.65	±0.86	±1.00
$r = 0.8$									0.00	±0.75	±1.00
$r = 0.9$										0.00	±1.00
$r = 1.0$											±0/0

Values of $u = \sqrt{\frac{1}{1-r}\left(1 - \frac{r}{r_0}\right)}$ for values of r and r_0 between 1 and ∞:

$r_0 =$	1.0	1.2	1.4	1.6	1.8	2.0	2.5	3.0	3.5	4.0	5.0	6.0	∞
$r = 1.0$	±0/0												
$r = 1.2$	±1.00	0.00											
$r = 1.4$	±1.00	±0.65	0.00										
$r = 1.6$	±1.00	±0.75	±0.49	0.00									
$r = 1.8$	±1.00	±0.79	±0.60	±0.40	0.00								
$r = 2.0$	±1.00	±0.82	±0.65	±0.50	±0.33	0.00							
$r = 2.5$	±1.00	±0.85	±0.72	±0.61	±0.51	±0.41	0.00						
$r = 3.0$	±1.00	±0.86	±0.75	±0.66	±0.57	±0.50	±0.32	0.00					
$r = 3.5$	±1.00	±0.88	±0.77	±0.68	±0.62	±0.55	±0.40	±0.26	0.00				
$r = 4.0$	±1.00	±0.88	±0.79	±0.71	±0.64	±0.57	±0.45	±0.33	±0.22	0.00			
$r = 5.0$	±1.00	±0.89	±0.80	±0.73	±0.66	±0.61	±0.50	±0.41	±0.33	±0.24	0.00		
$r = 6.0$	±1.00	±0.89	±0.81	±0.74	±0.68	±0.63	±0.53	±0.45	±0.37	±0.31	±0.20	0.00	
∞	±1.00	±0.91	±0.84	±0.79	±0.74	±0.71	±0.63	±0.57	±0.54	±0.50	±0.45	±0.41	0.00

The graphic representation of these numerical values in Figure 1 gives now a clear insight of the meaning of the result that arises from the formula, namely that the mutual acceleration of two particles *is infinite at the so-called critical distance ρ.* The latter called for diverse concerns against the general law of electric force that underlies the formula.

One indeed sees from the graphic representation that at distance ρ simultaneously with the infinite acceleration there occurs a jump of the relative velocity of both particles from $-c$ to $+c$, or the other way around, it occurs so abruptly that meanwhile the distance ρ does not change at all.

By becoming *infinite* the acceleration changes sign and as a consequence the velocity changes, in the same moment and without any loss of time, from $-c$ to $+c$, or vice versa. Before the distance ρ could experience even the smallest finite change, the transition of the velocity c to the opposite has already happened.

The formula above tells in principle only that there is an abrupt reflection of the particles from one another in the moment when they get to the distance ρ, just as the formula in mechanics for two colliding *elastic* balls, which also reflect from one another, the more abrupt, the smaller the balls and the larger their elasticity coefficient. *Instantaneous* reflection is the *limiting case*, which really does not happen, but which to date neither is declared odd or absurd by the principles of mechanics. Finally it deserves special attention that according to the formula, for $r = \rho$, the acceleration would be infinite, but that the case $r = \rho$ does not *really happen* just as does not exist an elastic body with infinite elasticity coefficient.

Further one sees that the totality of all curves representable according to the mentioned formula forms two groups completely separated from one another, namely, firstly, a group in which all distances of the particles are smaller than ρ and, secondly, a group in which they are larger than ρ. The two groups differ from one another in that, as shown above, the case in which the distance would be $= \rho$ neither really occurs, nor can it occur.

Both groups together fill out the whole space for all abscissa values from $r = 0$ to $r = \infty$ and for all ordinate values from $u = -c$ to $u = +c$, and none of these curves allows for an extension beyond the limits of this space. From this it follows that two electric particles, which are impelled only by mutual interaction and whose relative velocity is never larger than $+c$ and never smaller than $-c$, stay inside the mentioned limits.[41]

[41] [Note by WEW:] It would be different, if that case would happen that was excluded in the definition of work capability, namely that two electric particles would possess already initially a velocity $> +c$ or $< -c$, or if the two particles would move not only by mutual interaction, but in addition impelled by an *external* influence and thereby would have acquired such a velocity, either $> +c$ or $< -c$. Suppose such a case would really happen, then the motions of two such particles, if they would be impelled only by mutual interaction from this very moment on, would be represented by completely different curves which would be excluded from the region of the system of curves considered above. All these other curves would form a system closed in itself which would fill out the whole space. All curves of this second kind are represented in Figure 1 by dotted lines. Also these types of motion, or their representing curves, decompose into two groups separated from each other at the spot determined by the critical distance ρ. Namely, one group in which all distances of the particles are always smaller than ρ, and a group in which they are larger than ρ. Moreover, again there is a *complete symmetry* between the curve arms with ordinates $> +c$ and $< -c$. Further at $r = \rho$ both curve arms are connected with one another as a consequence of the abrupt change of the velocity from $\pm\infty$ to $\mp\infty$.

7 Electric Rays, Especially Reflexion and Scattering of Rays

The motion of two electric particles impelled only by mutual interaction, moving relative to one another both along their connecting line and orthogonal to it, have been considered in the *Electrodynamic Measurements*, Vol. X of these *Abhandlungen*, and for their determination the following equations have been found:[42,43]

$$\frac{u^2}{c^2} = \frac{r-r_0}{r-\rho}\left(\frac{\rho}{r_0}+\frac{r+r_0}{r}\cdot\frac{{\alpha_0}^2}{c^2}\right) , \tag{1}$$

$$r\alpha = r_0\alpha_0 , \tag{2}$$

where r is the distance of both particles from one another, and u and α their relative velocities in the direction of r and orthogonal to it; further r_0 denotes the value of r for which $u = 0$, α_0 the value of α, for which $r = r_0$, finally ρ the constant that depends on the nature and the masses ε and ε' of the two particles e and e', namely

$$\rho = 2\frac{\varepsilon+\varepsilon'}{\varepsilon\varepsilon'}\cdot\frac{ee'}{c^2} ,$$

where ρ is positive or negative like the product ee'. — If ρ shall have the significance of a distance of the two particles from each other, and which can *only be positive*, like r and r_0, then one has to set

$$\rho = \pm 2\frac{\varepsilon+\varepsilon'}{\varepsilon\varepsilon'}\cdot\frac{ee'}{c^2} ,$$

where the upper or lower signs is valid depending if the product ee' is positive or negative. Then for Equation (1) one has to set

$$\frac{u^2}{c^2} = \frac{r-r_0}{r\mp\rho}\left(\frac{r+r_0}{r}\cdot\frac{{\alpha_0}^2}{c^2}\pm\frac{\rho}{r_0}\right) ,$$

with the same determination of the signs. — Since hereinafter only electric particles of the *same type* will be considered,[44] always the upper signs will be valid. — The equation $r\alpha = r_0\alpha_0$ tells that $\alpha = 0$ for $r = \infty$ whenever

[42] [Note by HW:] Wilhelm Weber's *Werke*, Vol. IV, p. 273.

[43] [Note by AKTA:] [Web71, p. 273 of Weber's *Werke*] with English translation in [Web72, p. 124].

[44] [Note by AKTA:] That is, both positive or both negative.

r_0 and α_0 are given and finite, which is related to the existence of a straight asymptote with which the trajectory coincides at infinity.

Now we shall consider the case of two *equal* electric particles e and e' approaching each other with large, but as a consequence of mutual repulsion, decreasing velocity u whose largest value, namely for $r = \infty$, shall be denoted by u_0. For simplicity e shall be considered as stationary during this relative motion. Suppose that along the same trajectory as e' and with the same velocity relative to e there is a whole sequence of *equal* particles e'', e''', ... that follows e', moreover suppose they follow in such intervals that the mutual perturbations can be neglected.

From the law represented by the above Equation (1) there results the value of u for $r = \infty$, namely

$$u_0 = c\sqrt{\frac{{\alpha_0}^2}{c^2} + \frac{\rho}{r_0}}\ . \qquad (3)$$

Since now ρ is equal for equal particles, and also u_0 has been assumed to be equal, a difference can only occur with respect to the value of r_0 and the resulting value of α_0 according to Equation (3).

The system of all these particles is called an *electric ray*,[45] and the asymptote, in which the particles are located when they are very far away from e, serves to determine the *direction* of the ray.

If for all particles one had $r_0 = \rho[c^2/{u_0}^2]$, from which it would follow $\alpha_0 = 0$, then they all would move along the same line up to distance r_0, on which all of them would again go backward along the same line. But if α_0 is non-zero and, moreover, simultaneously with r_0 different for all particles e', e'', ..., for which u_0 is non-zero, but very small for all of them, then each particle will deviate from each asymptote as r approaches r_0. The angle which is then formed by the line, e.g. ee', as a consequence of the deviation, with the direction of the ray, shall be denoted by φ. Set $\varphi = \varphi_0$ if the decreasing distance between $e'e$ became $= r_0$, where indeed the velocity in direction $e'e$ is $= 0$ and in the orthogonal direction it is $= \alpha_0$.

From that moment on, where the distance becomes $r = r_0$, the two particles e and e' start moving away from each other. And their connecting line approaches another line which, with the direction that ee' had when it became $= r_0$, also forms an angle $= \varphi_0$ and with the direction of the original ray the angle $= 2\varphi_0$, which shall be called the *angle of reflexion.* But this angle of reflexion is very different for the different pairs of particles ee', ee'', ..., which belong to the same ray, according to the difference of the

[45] [Note by AKTA:] *Elektrischer Strahl* in the original.

values of α_0 or r_0/ρ, which tells that such a reflected ray also gets *scattered* simultaneously. This *scattering* of electric rays shall now be determined in more detail according to the previous laws.

To begin with from the above law (2) there results the growth of the angle φ, namely

$$d\varphi = \frac{\alpha\, dt}{r} = \frac{\alpha_0 r_0}{r^2}\, dt\ . \qquad (4)$$

Further substituting in Equation (1) the value of ${\alpha_0}^2 = {u_0}^2 - [\rho/r_0]\, c^2$ resulting from Equation (3), then one gets

$$\frac{u^2}{c^2} = \frac{dr^2}{c^2\, dt^2} = \frac{r - r_0}{r - \rho}\left(\frac{r + r_0}{r} \cdot \frac{{u_0}^2}{c^2} - \frac{\rho}{r}\right)\ , \qquad (5)$$

consequently, if r decreases with increasing t,

$$dt = -dr\sqrt{\frac{r(r-\rho)}{(r-r_0)({u_0}^2(r+r_0) - \rho c^2)}}\ . \qquad (6)$$

From this it follows

$$d\varphi = \frac{\alpha_0 r_0}{r^2}\, dt = -\frac{\alpha_0 r_0}{u_0} \cdot \frac{dr\sqrt{r^2 - r\rho}}{r^2\sqrt{r^2 - \frac{c^2}{{u_0}^2}\,\rho r - \left({r_0}^2 - \frac{c^2}{{u_0}^2}\,\rho r_0\right)}}\ ,$$

or, if one sets $1/r = s$,

$$d\varphi = +\frac{\alpha_0 r_0}{u_0} \cdot ds\sqrt{\frac{1 - \rho s}{1 - \frac{c^2}{{u_0}^2}\,\rho s - \left({r_0}^2 - \frac{c^2}{{u_0}^2}\,\rho r_0\right) s^2}}\ , \qquad (7)$$

from which one sees that φ can be represented by *elliptic functions*.

If one restricts now attention to those cases where the value of ${\alpha_0}^2 = {u_0}^2 - [\rho/r_0]\, c^2$, hence also the value of $({r_0}^2 - [c^2/{u_0}^2]\,\rho r_0)s^2$, *either* vanishes completely or is yet very small, then the above equation reduces in the *former* case to

$$d\varphi = +\frac{\alpha_0 r_0}{u_0} \cdot ds\sqrt{\frac{1 - \rho s}{1 - r_0 s}}\ , \qquad (8)$$

and in the *latter* case, where α_0 is supposed to be very small although not vanishing completely, set $r_0\,[{\alpha_0}^2/{u_0}^2] = r_0 - [c^2/{u_0}^2]\,\rho = \beta$ and suppose β is so small that in Equation (7), which via introduction of β turns into

$$d\varphi = \frac{\alpha_0 r_0}{u_0} \cdot ds\sqrt{\frac{1 - \rho s}{(1 - r_0 s)(1 + \beta s)}}\ ,$$

one can write $(1 - \frac{1}{2}\beta s)$ in place of the factor $\sqrt{1/(1+\beta s)}$ whereby Equation (7) turns into

$$d\varphi = \frac{\alpha_0 r_0}{u_0} \cdot \left(1 - \frac{1}{2}\beta s\right) ds \sqrt{\frac{1-\rho s}{1 - r_0 s}} \,, \tag{9}$$

from where, if one sets $S = 1 - (\rho + r_0)s + \rho r_0 s^2$, it results

$$\int d\varphi = \frac{\alpha_0 r_0}{u_0} \left[\int \frac{ds}{\sqrt{S}} - \left(\frac{1}{2}\beta + \rho\right) \int \frac{s\, ds}{\sqrt{S}} + \frac{1}{2}\beta\rho \int \frac{s^2\, ds}{\sqrt{S}} \right] .$$

If one sets $b = -(\rho + r_0)$ and $c = \rho r_0$, then carrying out the integration one gets:[46]

$$\frac{u_0}{\alpha_0 r_0} \int d\varphi =$$

$$\left[1 + \frac{b}{4c}(\beta + 2\rho) + \frac{\beta\rho}{4c}\left(\frac{3b^2}{4c} - 1\right)\right] \frac{1}{\sqrt{c}} \cdot \log\left(\sqrt{S} + s\sqrt{c} + \frac{b}{2\sqrt{c}}\right)$$

$$-\frac{1}{c}\left(\rho + \frac{\beta}{2}\left(1 + \frac{3b}{4c}\rho\right) - \frac{\beta\rho}{4}s\right)\sqrt{S} \,,$$

from which, setting $m = \rho/r_0$, $n = c/u_0$ and so $\alpha_0/u_0 = \sqrt{1 - mn^2}$ according to Equation (3), one gets:

$$\varphi_0 = \int_{s=0}^{s=\frac{1}{r_0}} d\varphi =$$

$$\sqrt{1-mn^2} \cdot \left[\frac{1-m}{2}\left(1 - \frac{1+3m}{8m}(1-mn^2)\right)\sqrt{\frac{1}{m}} \cdot \log\frac{1+\sqrt{m}}{1-\sqrt{m}}\right.$$

[46] [Note by WEW:] Namely

$$\begin{aligned}
\int \frac{ds}{\sqrt{S}} &= \frac{1}{\sqrt{c}} \cdot \log\left(\sqrt{S} + s\sqrt{c} + \frac{b}{2\sqrt{c}}\right) \,, \\
\int \frac{s\, ds}{\sqrt{S}} &= -\frac{b}{2c\sqrt{c}} \cdot \log\left(\sqrt{S} + s\sqrt{c} + \frac{b}{2\sqrt{c}}\right) + \frac{\sqrt{S}}{c} \,, \\
\int \frac{s^2\, ds}{\sqrt{S}} &= \frac{3b^2 - 4c}{8c^2\sqrt{c}} \cdot \log\left(\sqrt{S} + s\sqrt{c} + \frac{b}{2\sqrt{c}}\right) + \frac{1}{2c}\left(s - \frac{3b}{2c}\right)\sqrt{s} \,.
\end{aligned}$$

$$+ 1 + \frac{1-3m}{8m}(1-mn^2)\Big] \ . \qquad (10)$$

Based on this the following Table of values of φ_0, for different values of m and n, has been calculated:[47]

	$n = 1$	$n = 2$
$m = 1$	0	
$m = 1/2$	0.9658	
$m = 1/3$	1.1269	
$m = 1/4$	1.1479	0
$m = 1/5$	1.2272	0.7776
$m = 1/6$	1.2486	0.9688
$m = 1/7$	1.2629	1.0690
$m = 1/8$	1.2732	1.1302
$m = 0$	1.3750 (see footnote)	1.3750 (see footnote)

From here it results for all particles e', e'', ... of an electric ray which *approach* the particle e from large distance with velocity u_0, that once they have reached the distance r_0 they turn around and move away again from e with a velocity which again increases to u_0 at large distance, that however the two directions, in which the two particles first approached each other with velocity u_0 and then turned backward, form an angle $2\varphi_0$ which for the different pairs is very different according to the difference of the value of r_0.

The diversity of the angle $2\varphi_0$, which was called the *angle of reflexion*, for the different pairs of particles according to the different values of r_0, forms the fact which is given the name *scattering* of electric rays by reflexion. Moreover, the discovered law of dependence of the angle of reflexion $2\varphi_0$ from m and n gives a precise determination of this scattering, if one takes into account the following, that n has for all particles of one and the same ray the same value, which depends on u_0 according to the equation $n = c/u_0$. Further, that for each pair of particles, m can be determined for any distance r — using the three equations $m = \rho/r_0$, ${\alpha_0}^2 = {u_0}^2 - [\rho/r_0]\, c^2$ and $\alpha_0 r_0 = \alpha r$, after elimination of r_0 and α_0 — from the relative velocity α of the two particles in the direction orthogonal to their connecting line, namely via the equation:

$$m^2 + \frac{\rho^2 c^2}{r^2\alpha^2} m = \frac{\rho^2 {u_0}^2}{r^2\alpha^2} \ .$$

[47][Note by HW:] The itemized values 1.2500 in the original Memoir for $m = 0$, $n = 1$ and $n = 2$ have later been changed by W. Weber to the ones shown in the Table.

8 Application of the Theory of Reflection and Scattering of Electric Rays to the Luminiferous Ether and to Gases According to the Krönig-Clausius' Theory of Molecular Collisions

The reflection and scattering of electric rays, which are composed of electrically identical pairs of particles, which move in space approaching or moving away from each other with the same velocity, leads to a similar aggregate state of the whole system of such particles in empty space as is assigned to gases in the gas theory according to Krönig and Clausius,[48] only with the difference that the moving particles of gases are ponderable particles, while those electric particles are commonly called imponderable, because the validity of the general law of gravitation for them is so far at least not proved. Only according to Mossotti's theory of gravitation (see Zöllner, *Wissenschaftliche Abhandlungen*, Vol. 1, no. 2, Leipzig 1878),[49] according to which all gravitational forces result from electric repulsive and attracting forces, would all interactions, of ponderable and imponderable particles, be subjected to common rules in that every ponderable particle would be an *electric double particle* (like a double star), namely a positive and a negative electric particle, which rotate around one another.

According to Mossotti's conception it results by itself that, if these particles move in empty space, as one assumes in Krönig-Clausius' theory of gases, then there would result from the laws of electric interaction for these in empty space moving particles similar laws of reflection and scattering, as those found in the previous Section for moving identical electric particles, as is easily seen, if one pays attention that those laws are valid preferably for pairs of identical particles which approximate each up to a distance r_0 which has ρ as a lower bound. Because two ponderable molecules contain two pairs of identical electric particles,[50] and for each of these pairs there is a distance ρ up to which the particles of the pair can get, since their repulsive force would become infinite, which only is going to be obstructed in that the two particles are brought to a standstill by the ever-increasing repulsive

[48] [Note by AKTA:] [Krö56], [Cla57b] with English translation in [Cla57a], and [Cla79].

[49] [Note by AKTA:] [Mos36] with English translation in [Mos66]; [Zöl78] and [Zöl82]. Weber wrote Mossotti's name as Mosotti. We corrected this misprint.

[50] [Note by AKTA:] Each ponderable molecule would be composed of a two particles orbiting around one another. These particles would have opposite electric charges of the same magnitude. See [Web94, Section 1] with English translation in [Web08, Section 1].

force, indeed before they get to the distance ρ, on which they will again move away from each other due to the continuing repulsive force caused by their interaction, just as they had moved toward each other before.

According to this, the laws of reflection and scattering for rays of identical electric particles found in the former Section can be transferred to rays of ponderable molecules, composed according to Mossotti's conception. And if these ponderable molecules are gas molecules, then they form an aggregate state of the gas, which corresponds completely to the aggregate state assigned to gases by the theory of Krönig-Clausius, without it being necessary to assign to these ponderable gas molecules with Krönig a special form and elasticity, or with Clausius and Maxwell[51] special repulsive forces inversely proportional to a higher power of distance.

But if there exists a space, e.g. celestial space, in which there are no ponderable molecules at all, then the possibility suggests itself that this space contains the particles of one of the two constituents of these ponderable molecules, i.e. either the positive or the negative electric particles. Being in motion, these would also form a body of a special aggregate state which however, since it consists only of identical electric particles, could not be called a ponderable body, but an imponderable *ether*. For which, however, it would also be valid the laws of motion developed by Maxwell (*Philos. Transact.* 1867) for a *dynamic media*, in particular, the laws of *wave propagation* in accordance with the laws of propagation of light waves would apply. Such conception of a space filling medium, composed of mutually repelling particles, seems to be possible without fixed spatial borders only under the hypothesis of an infinite extension all the way to infinity, however it seems that restricting such a medium to a finite space without fixed borders is possible according to Mossotti, because this medium surrounds a ponderable body of Mossotti's type, which would act attracting the medium and thereby holding it together.

9 Laws of Motion of Two Electric Particles Impelled by Mutual Interaction and *External* Influence

It shall be considered only the simple case where the *external* influence on particle e consists of a constant force in the direction of the prolonged line $e'e$, which divided by the sum $\varepsilon + m$ of the own mass of the particle e and the

[51][Note by AKTA:] [Max67] and [Max65].

ponderable mass tightly connected to it, provides the quotient g.[52] — Let the *external* influence on the other particle e' consist of a force which is *oppositely equal* to the force that acts on e' as a result of the *mutual interaction* of e and e'.

According to Section 5 the *potential* V of the two particles e and e' at distance r — understanding V as that function whose differential quotient dV/dr represents the *repulsive force* — is given by

$$V = \frac{ee'}{r}\left(\frac{dr^2}{c^2 \cdot dt^2} - 1\right) .$$

From this now follows the *acceleration through interaction* of the particle e with respect to r, namely $= [1/(\varepsilon + m)] \cdot [dV/dr]$, and that of the particle e' in the opposite direction, namely $= [1/\varepsilon'] \cdot [dV/dr]$, from where it results the relative acceleration of the two particles *through interaction*:

$$= \left(\frac{1}{\varepsilon + m} + \frac{1}{\varepsilon'}\right) \cdot \frac{dV}{dr} .$$

In addition to this, it is to be taken into account the acceleration *due to external influence*, which for e is $= g$ in the direction r, and for e' in the same direction is $= [1/\varepsilon'] \cdot [dV/dr]$, from where it results the *relative* acceleration of the two particles *due to external influence*:

$$= g - \frac{1}{\varepsilon'} \cdot \frac{dV}{dr} .$$

The total relative acceleration is obtained from this as follows:

$$\frac{d^2r}{dt^2} = \frac{1}{\varepsilon + m} \cdot \frac{dV}{dr} + g .$$

Multiplying this equation by $2dr$ one obtains

$$2\frac{dr}{dt} \cdot \frac{d^2r}{dt^2} = \frac{2}{\varepsilon + m} \cdot \frac{dV}{dr} + 2g\, dr ,$$

and from here by integration from $r = r_0$ to $r = r$, where r_0 denotes the value of r at the time where $dr/dt = 0$, one gets:

$$\frac{dr^2}{dt^2} = \frac{2}{\varepsilon + m}\left[\frac{ee'}{r}\left(\frac{dr^2}{c^2 \cdot dt^2} - 1\right) + \frac{ee'}{r_0}\right] + 2g(r - r_0) .$$

[52][Note by AKTA:] Consider a particle with charge e and mass ε connected to a ponderable mass m. If there is a force F acting on this system, then according to Newton's second law of motion this system will move with acceleration $g = F/(\varepsilon + m)$ relative to an inertial frame of reference.

Denoting $dr/c\,dt$ by u and taking into account that $\pm[ee'/\rho] = a = \frac{1}{2}[\varepsilon\varepsilon'/(\varepsilon+\varepsilon')]c^2$, then one gets

$$u^2 = \pm\frac{\varepsilon\varepsilon'\rho}{(\varepsilon+m)(\varepsilon+\varepsilon')}\left(\frac{1}{r_0} - \frac{1}{r} + \frac{1}{r}u^2\right) + \frac{2g}{c^2}(r - r_0)\ ,$$

and from here

$$u^2 = \frac{\pm\frac{\varepsilon\varepsilon'\rho}{(\varepsilon+m)(\varepsilon+\varepsilon')}\left(\frac{1}{r_0} - \frac{1}{r}\right) + \frac{2g}{c^2}(r - r_0)}{1 \mp \frac{\varepsilon\varepsilon'\rho}{(\varepsilon+m)(\varepsilon+\varepsilon')}\cdot\frac{1}{r}}\ .$$

Setting now $\varepsilon\varepsilon' \cdot [\rho/(\varepsilon+m)(\varepsilon+\varepsilon')] = \rho'$, then it results:

for positive values of ee', $$u^2 = \frac{\rho'}{\rho' - r}\left(1 - \frac{r}{r_0}\right)\left(1 + \frac{2g}{c^2}\cdot\frac{rr_0}{\rho'}\right)\ , \qquad (1)$$

for negative values of ee', $$u^2 = \frac{\rho'}{\rho' + r}\left(1 - \frac{r}{r_0}\right)\left(1 - \frac{2g}{c^2}\cdot\frac{rr_0}{\rho'}\right)\ , \qquad (2)$$

or, if one expresses r and r_0 as parts of ρ':

for positive values of ee', $$u^2 = \frac{1}{1 - r}\left(1 - \frac{r}{r_0}\right)\left(1 + \frac{2g\rho'}{c^2}\cdot rr_0\right)\ , \qquad (3)$$

for negative values of ee', $$u^2 = \frac{1}{1 + r}\left(1 - \frac{r}{r_0}\right)\left(1 - \frac{2g\rho'}{c^2}\cdot rr_0\right)\ . \qquad (4)$$

From here it results, as one sees by setting $g = 0$, the equations found in Section 6 for two particles impelled only *through mutual interaction*. If, in contrast, one sets $\varepsilon = 0$ or $\varepsilon' = 0$, in which case $\rho' = 0$, then one obtains from (1) and (2) the relation

$$u^2c^2 = \frac{dr^2}{dt^2} = 2g(r - r_0)\ ,$$

i.e. the *law of the free fall*, where $(r - r_0)$ denotes the *fall space*.

The *graphic* representation of these types of motion of a pair of electric particles impelled by *interaction* and *external influence* easily results for positive values of ee' from the graphic representation in Figure 1 of the types of motion of a pair of particles impelled *only by interaction*, without external influence; one only needs to enlarge in Figure 1, with unchanged abscissas r, all ordinates $\pm u$ of any of the curves determined by a certain value of the constant, by the ratio $1\colon \sqrt{1 + [2g\rho'/c^2]\, r_0 r}$ in order to get that particular curve which represents the motion of the pair of particles under the given *external* influences, where the only thing to be noted, is that r_0 and r are represented as parts of ρ', instead of ρ, and that $\rho'\colon \rho = [\varepsilon\varepsilon'/(\varepsilon + \varepsilon')]\colon \varepsilon + m$, i.e. it behaves nearly $= \varepsilon\colon m$ for small values of ε.

One sees from this that, also with *external influence* as the one mentioned above, for a *certain distance* ρ' of identical particles from one another, their relative acceleration due to mutual interaction, though infinite according to the formula, however this distance ρ' could never occur by the reason mentioned in the former Section which is also valid here. Indeed, if the case would really happen where the distance of the particles would be $= \rho'$, then it would have to happen in that either both particles *approach* or *move away* from each other. If now during approach there is *repulsion*, during departure *attraction*, and if that repulsion and this attraction grow according to the mentioned law in such a way that it would become infinite for the distance ρ', then in none of the two cases the particles will reach the distance ρ', but are forced to stop and to return before they get there. This happens according to our law and it becomes clear the reason why the case of infinite acceleration really can never happen according to this law.

Hence also the case discussed in the present Section of the combination of *mutual interaction* with *external* influence does not offer a reason against the established law. Therefore it is not necessary at all, in order to defend this law to seek refuge in the hypothesis that, as ρ and ρ' are molecular distances, *special molecular forces* could yet come into consideration, as a consequence of which the case of infinite acceleration in the molecular distances ρ or ρ' would be eliminated.

By the way the distances ρ and ρ' always stay molecular distances, because, although ρ can be increased by increasing the mutually interacting electric masses, at least one of the two mutually interacting electric masses will be bound to a *ponderable* mass, which must be moved on with it, as in the just considered case, whereby a *reduction* of ρ takes place, in the ratio of the total mass $\varepsilon + m$ to the electric [mass] ε, where m denotes a ponderable mass in relation to which ε vanishes.

The case considered in the present Section of two electric particles impelled by mutual interaction and *external* influence is the one, to which the objection raised by Helmholtz referred to. That objection was submitted to a closer inspection by Neumann in his Memoir, p. 91 and the following of the present Volume.[53]

Neumann saw, p. 92, in the "absurd result of infinite acceleration" criti-

[53] [Note by AKTA:] [Neu74].

cized by Helmholtz[54] at the so-called critical distance, a new argument that my electric fundamental law would require (similar to Newton's) a certain modification for extraordinary small distances. Meanwhile he raised the objection that this case of infinite acceleration could be arranged such that only *large* distances come into consideration which would exclude a modification of the result by molecular forces.

Neumann remarks that for these cases, in which only large values of critical distances come into consideration, neither *reality* nor *feasibility* have been proved, without which these cases can not be used as a test of a physical law, and that Helmholtz' objection would not gain *serious significance* before that proof had been provided.

In contrast, in the present and in the previous Section, it has been provided the proof, that the *possibility of the case* in which according to Helmholtz the "absurd result of infinite acceleration" would occur will be excluded *by means of the fact* that the two particles, before they can get to the critical distance, must have approached each other beforehand, either from *smaller* or from *larger* distance. But because of the, upon getting closer, *backwards* acceleration increasing to infinity, i.e. deceleration, which happens when the particles approach both from *smaller* and from *larger* distance, they *can never reach* the critical distance. From this it follows, that the "absurd result of infinite accelerations" criticized by Helmholtz *does not exist at all*, and that only an error made by Helmholtz and so far not disproved has led there. — The magnitude of the critical distance is completely irrelevant here. —

However, in the next two Sections there shall be discussed some cases, in which it was believed that a significant enlargement of the so-called critical distance can be obtained, and which have attracted special interest through the related conclusions.

[54][Note by WEW:] An *infinite acceleration* occurs frequently when considering colliding bodies and is not considered absurd in mechanics, but as a *limiting case* of growing elasticity. If this *limiting case* never happens, then the same is valid in our present case, as will be shown immediately.

10 Laws of Motion of an Electric Particle Inside an *Electrified Spherical Shell* that is Impelled by Electric Interaction and *External* Influence

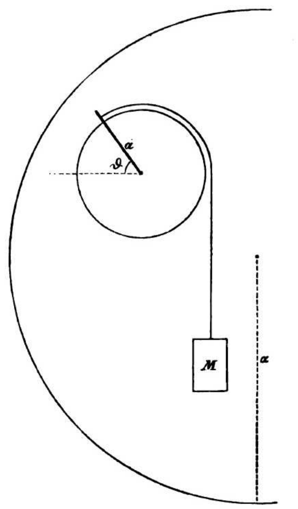
In these *Abhandlungen*, pp. 103–106 of this Volume,[55] C. Neumann has particularly highlighted and discussed the following case:

> "Let it be given a fixed spherical shell (of radius α) uniformly covered with electricity. In the interior of this shell let there be a cylinder covered with electricity (of radius $= a$ and moment of inertia $= \mathfrak{M}$), rotatable about its firmly situated horizontal axis. Let around this cylinder be wound up a thread at whose free end is fixed a weight Mg. — The goal is to examine more closely the motion achieved by the cylinder under the influence of the electrified spherical shell on the one hand, and under the influence of the weight Mg on the other hand.
>
> Hereby let us suppose that the cylinder is connected *rigidly* and *indissolubly* with its existing electric matter, and that the same also takes place for the spherical shell."[56]

Neumann arrived for this case on page 106, assuming hereby as *constant*

[55][Note by AKTA:] [Neu74, pp. 103-106].

[56][Note by AKTA:] Neumann presented the following Figure to describe this configuration:



the electric charges of the cylinder and the spherical shell, to the following equation:

$$L\vartheta'^2 = Mga\vartheta + \text{constant} \ ,$$

or, differentiated with respect to t,

$$2L\vartheta'' = Mga \ ,$$

where ϑ denotes the rotation angle, ϑ' the rotational velocity, and ϑ'' the rotational acceleration of the cylinder, and, when H denotes the [surface] density of the electricity on the spherical shell and Σe the charge of the cylinder surface, one sets

$$L = \frac{Ma^2 + \mathfrak{M}}{2} - \frac{4\pi\alpha H \cdot a^2 \Sigma e}{3c^2}$$

and supposes that it is *constant*.

Neumann linked to this the following statement:

> "If the *constant* $L =$ positive, then the attached weight Mg will *sink* with accelerated velocity. If $L = 0$ there originates an infinitely large acceleration. If $L =$ negative any weight will be lifted up with accelerated velocity. In this last case the weight could, if one supposes an infinitely long threat, be lifted infinitely high, hence an infinitely large work be done.
>
> However, if one investigates if the cases $L = 0$ and $L =$ negative can really occur, then one encounters the same difficulties as earlier." — —

Of these difficulties, indicated here by Neumann, the following two shall be especially emphasized, namely *firstly* those that arise from the limits posed on the electric separating forces[57] through the *nature of bodies*, *secondly* the difficulties linked with the assumption of a *constant value of L*, which consist in the fact that this hypothesis is linked to the assumption of *certain unchangeable charges on the spherical shell and on the cylinder.*

1. Difficulties Arising from the Limits Imposed on the Electric Separating Forces through the Nature of the Bodies

Denoting the electricity of the cylinder and the spherical shell shortly by e and e' (instead of Σe and $4\pi^2\alpha H$), and setting the moment of inertia of the

[57] [Note by AKTA:] *Elektrischen Scheidungskräften* in the original. This expression can also be translated as "electric force of separation".

cylinder $\mathfrak{M} = ma^2$, then

$$L = \left(\frac{M+m}{2} - \frac{ee'}{3\alpha c^2} \right) a^2 \ ;$$

consequently for $L = 0$:

$$ee' = \frac{3}{2}\alpha c^2 \left(M + m\right) \ .$$

Now $2e'/\alpha$ is the required *separating force* [acting] on the charge e' of a spherical shell;[58] however the magnitude of this *separating force* is limited and depends on the *separating means* present in nature, because although the variety of these means is huge there is yet no means for [generating] *infinitely large separating forces.*

If now $L = 0$, then according to the equation above it must be valid

$$\frac{2e'}{\alpha} = 3c^2 \cdot \frac{M+m}{e} \ ,$$

[58][Note by WEW:] The separating force exerted by a spherical shell of radius a covered uniformly with electricity e' acting on an *external* linear conductor of unlimited length ℓ, which lies in the extension of a radius, is the *difference* of the *repulsive force* $= e' \int_0^\ell \frac{dx}{(\alpha+x)^2}$, exerted on the unit *positive* electricity contained in each unit length interval of the conductor, and the *attractive force* $= -e' \int_0^\ell \frac{dx}{(\alpha+x)^2}$, exerted on the unit *negative* electricity contained in each unit length interval of the conductor; consequently

$$= 2e' \int_0^\ell \frac{dx}{(\alpha + x)^2} = 2e' \left(\frac{1}{\alpha} - \frac{1}{\alpha + \ell} \right) \ ,$$

from where for unlimited value of ℓ it results the *separating force* $= 2e'/\alpha$, as stated above.

If in this conductor a pillar is interposed, through which the charge on the sphere remains stationary, then this proves that the separating forces exerted by the charge on the sphere and by the pillar on the conductor are equal and opposite to each other, whereby also the separating force of the pillar is determined, namely $-2e'/\alpha$.

But it is also clear that, when the spherical shell was not yet charged, it would get charged from the pillar, and that this charge would grow, until it got $= e'$, assuming a sphere of radius $= \alpha$, i.e. until the *separating force of the charge on the sphere* would have become $= 2e'/\alpha$ and *cancelled the separating force of the pillar.*

Further from here it follows that two spherical shells with charges e' and ne', whose radii are α and $n\alpha$, whose *potentials* for all points in the interior, namely e'/α and $ne'/n\alpha$, are consequently equal, could be connected by a conductor, without any part of the charge going from one shell to the other, in conformity with the theorem, that in the case of the equality of the *potentials* in the interior of two conductors there occurs no transfer of electricity. —

It is yet to be remarked, that the above *separating forces* are expressed in *mechanical measure* and are to be multiplied by $155\,370 \cdot 10^6 = [c/2\sqrt{2}]$, in order to express them in *magnetic measure.*

or, since $c = 439\,450 \cdot 10^6$, it must be true

$$\frac{2e'}{\alpha} \cdot \frac{e}{2(M+m)} = 289\,670 \cdot 10^{18} \ .$$

— It should be hardly possible to keep a charged cylinder, with fixed axis of rotation, whose charge e were in *absolute measure* larger than the ponderable mass $2m$ expressed *in milligram*; but adding to $2m$ yet the double mass of the weight $= 2M$, one can surely assume $e/2(M+m)$ being a *proper* fraction. From which it follows that to charge the spherical shell, in case $L = 0$ occurred, it would be necessary a separating force which *in mechanical measure* should be $= 2e'/\alpha > 289\,670 \cdot 10^{18}$, i.e. a separating force which superseded at least 261 *trillion* times the largest of the measured ones in the *Electrodynamic Measurements*, Vol. V of these *Abhandlungen*, pp. 243-250,[59,60] namely $= 2\frac{6410.5}{11.567} = 1108$. That there exist yet unknown bodies in nature which permit the possibility for such large separating forces may be justly doubted. Enlargement of the two coefficients $e/(M+m)$ and e'/α by a factor 10 or 100 would not help at all; if the *nature of bodies* does not allow to enlarge the product of these two coefficients by many *trillion* times, then the production of the case $L = 0$ remains always *impossible*.

But also if in the nature of bodies the possibility of such large separating forces would exist, then even with such separating forces the required charges could not be produced, because there does not exist an *insulator*, stable enough to resist the expansive forces of such charges, which would explode stronger than gunpowder charges and destroy everything.

But even if such stable and perfect insulators would exist, which could even resist the tremendous expansive forces of such charges, and supposing one could bring the charge of the sphere to the required magnitude, then one would have achieved $L = 0$, but even then the acceleration ϑ'' would *not become infinite*, also not according to the law underlying the above calculation, as shall be proved in the following Section.

[59] [Note by HW:] Wilhelm Weber's *Werke*, Vol. III, pp 631-638.

[60] [Note by AKTA:] [KW57, Sections 9-11, pp. 631-638 of Weber's *Werke*] with English translation in [KW19, Sections 9-11, pp. 28-36].

11 Continuation

2. Difficulties Linked to the Assumption of a Constant Value of L

Refraining from the doubt explained in the previous Section, if, under the barriers imposed on the *electric separating forces* by the *nature of bodies*, [the case] $L = 0$ could actually occur, yet it remains open to discuss the question, if the precisely determined product of the two charges e and e' could be *kept constant* for $L = 0$, which must be assumed if L shall be *constant*, in fact $= 0$. Further, there is the question which influence it would have, if one of the two charges was *variable*.

The value of L depends on the charges e and e' of the cylinder and the spherical shell, more precisely, for the value of $L = 0$ to be *constant*, it is not only relevant the *magnitude* of the charges e and e', but also the *manufacturing method* of an exactly determined value.

If in addition one of the two charges, namely the cylinder charge e, would remain *constant*, yet the charge of the spherical shell e' would have to remain *variable*, in order to get via its steady growth to the point in which $L = 0$ would take place. But even then the charge e' would not abruptly stop changing, to stay from now on *completely constant*, but without doubt it would always oscillate within certain limits, because the production of the required charge *with absolute precision* is not possible at all, but only within certain wider or narrower limits. Consequently the charge e' would always have to be considered as a function of time, which for any short time interval can be represented by $e' = p + qt$.

Now *in this case, in which* e' *is variable,* it is valid the equation established by Neumann on p. 105 of this Volume,[61,62] namely

$$T = P - U + Mga\vartheta + \text{constant} \ ,$$

where $T = [(M + m)/2] \cdot a^2\vartheta'^2$, $P = [ee'/3\alpha c^2] \cdot a^2\vartheta'^2 + \text{constant}$, and $U = ee'/\alpha$, which therefore changes [its magnitude] simultaneously with e'.

Setting now $e' = p + qt$ and $L = ([(M + m)/2] - [ee'/3\alpha c^2])\, a^2$, then one gets

$$L\vartheta'^2 = Mga\vartheta - \frac{e}{\alpha}(p + qt) + \text{constant} \ ,$$

or, differentiated with respect to t,

$$2L\vartheta'' = Mga - \frac{eq}{\alpha\vartheta'} \ ,$$

[61][Note by HW:] *Abhandlungen der Königl. Sächs. Gesellschaft der Wissenschaften*, mathematisch-physische Klasse, Vol. 11.

[62][Note by AKTA:] [Neu74, p. 105].

from where for $L = 0$ it follows that *either* $\vartheta'' = \infty$, *or* (if ϑ'' is not infinite), $\vartheta' = eq/Mga\alpha$.

Now this alternative will be decided, if one takes into account, that L varies with time t. One calculates the time t from that instant, where according to the equation $e' = p + qt$ one would have $L = 0$, from where it follows

$$\frac{M+m}{2} = \frac{ep}{3\alpha c^2} \ .$$

It is then, after the time element δ,

$$L = -\frac{eq\delta}{3\alpha c^2}\, a^2 \ ,$$

consequently, if one puts this value for L in the above equation,

$$2L\vartheta'' = -\frac{2eq\delta}{3\alpha c^2}\, a^2 \cdot \vartheta'' = Mga - \frac{eq}{\alpha\vartheta'} \ .$$

Since now for $L = 0$ and $t = 0$ at a finite value of ϑ'' it was found that

$$\vartheta' = \frac{eq}{Mga\alpha} \ ,$$

and the value of ϑ' for $t = \delta$, if δ is *vanishingly small*, is not noticeably different from the value of ϑ'' for $t = 0$, thus it yields:

$$2L\vartheta'' = -\frac{2eq\delta}{3\alpha c^2}\, a^2 \cdot \vartheta'' = Mga - \frac{eq}{\alpha} \cdot \frac{Mga\alpha}{eq} = 0 \ ,$$

according to which $\vartheta'' = 0$.

Now this value $\vartheta'' = 0$ is valid, no matter how small q may be, it shall consequently also apply, when $q = 0$.

One sees from this, whenever the case $L = 0$ takes place, only the transition is from smaller values to larger ones or vice versa, so that the acceleration ϑ'' for $L = 0$ is not at all infinite, but $= 0$, which eliminates all objections that are based on the claimed *infinite acceleration*.

12 Conclusion

Even before Neumann noticed and investigated the case considered in the previous Section, Helmholtz had already called attention to a similar case, namely where an electric mass point ε is located in the *interior* of an electrified spherical shell, and he found the surprisingly simple result that the *components of the force exerted on ε by the electrified spherical shell are equal to the acceleration x'', y'', z'' multiplied by a constant factor.*

Further Helmholtz (Borchardt's *Journal*, Vol. 75)[63] developed from the fundamental law of electric interaction the equation of the vis viva, which results for the case of just *one* mass point μ with the electric quantum ε that moves in some space, which is limited by a spherical shell of radius R uniformly covered with electricity, namely the equation:[64,65,66]

$$\frac{1}{2}\left(\mu - \frac{8\pi}{3c^2} \cdot R\varepsilon\varepsilon'\right) q^2 - V + C = 0 ,$$

where ε' denotes the quantum of electricity per unit area on the spherical shell, q the velocity of the mass point μ on its trajectory s, hence $q = ds/dt$, and V the potential of the *non-electric* forces. It results from this equation via differentiation with respect to s:

$$\mu q \frac{dq}{ds} - \left(\frac{8\pi}{3c^2} \cdot R\varepsilon\varepsilon' \cdot q\frac{dq}{ds} + \frac{dV}{ds}\right) = 0 ,$$

where $[8\pi/3c^2] \cdot R\varepsilon\varepsilon' \cdot q[dq/ds]$ is the *electric* force exerted on μ along the direction of the trajectory s, and dV/ds the *non-electric* force exerted on μ in the same direction.

Since now $q = ds/dt$ denotes the velocity of the point μ on its trajectory s and $q[dq/ds] = dq/dt = d^2s/dt^2$ the acceleration of μ on its trajectory, it results, that the *electric* force $[8\pi/3c^2]\cdot R\varepsilon\varepsilon'\cdot q[dq/ds]$ exerted on μ found from the equation above, is the product of this acceleration $q[dq/ds]$ multiplied by the constant factor $[8\pi/3c^2] \cdot R\varepsilon\varepsilon'$, completely in agreement with the result stated above.

If now the *electric* force exerted on μ is proportional to the acceleration $q[dq/ds]$ of the point μ, on which *two* forces are exerted, namely beyond the stated *electric* force $[8\pi/3c^2] \cdot R\varepsilon\varepsilon' \cdot q[dq/ds]$, also the *non-electric* force dV/ds, then it is clear that $q[dq/ds]$ is obtained by dividing the sum of these *two* forces by μ, namely:

$$q\frac{dq}{ds} = \frac{\frac{8\pi}{3c^2} \cdot R\varepsilon\varepsilon' \cdot q\frac{dq}{ds} + \frac{dV}{ds}}{\mu} ,$$

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[63] [Note by AKTA:] [Hel73].

[64] [Note by WEW:] The factor $\frac{1}{2}q^2$ in the above equation is not $(\mu - [4\pi/3c^2]R\varepsilon e)$, as Helmholtz stated, but $(\mu - [8\pi/3c^2]R\varepsilon\varepsilon')$. Cf. Neumann, §§3 and 7 of his Memoir in this Volume.

[65] [Note by HW:] *Abhandlungen bei der Begründung der Königl. Sächs. Gesellschaft der Wissenschaften*, mathematisch-physische Klasse, Vol. 11.

[66] [Note by AKTA:] [Neu74, §§3 and 7].

from where it is found

$$q\frac{dq}{ds} = \frac{\frac{1}{\mu}\cdot\frac{dV}{ds}}{1-\frac{1}{\mu}\cdot\frac{8\pi}{3c^2}\cdot R\varepsilon\varepsilon'} \; .$$

Substituting this value for $q[dq/ds]$ in the expression of the *electric* force exerted on μ, then one obtains an expression for this force independent of the acceleration, namely

$$\frac{8\pi}{3c^2}\cdot R\varepsilon\varepsilon'\cdot q\frac{dq}{ds} = \frac{\frac{8\pi}{3c^2}\cdot R\varepsilon\varepsilon'}{\mu-\frac{8\pi}{3c^2}\cdot R\varepsilon\varepsilon'}\cdot\frac{dV}{ds} \; ,$$

from where one easily recognizes, that this *electric* force in the trajectory s is directed towards the same side, as the *non-electric* force dV/ds, as long as $[8\pi/3c^2]\cdot R\varepsilon\varepsilon' < \mu$, but that it has the opposite direction, as soon as $[8\pi/3c^2]\cdot R\varepsilon\varepsilon' > \mu$. But while $[8\pi/3c^2]\cdot R\varepsilon\varepsilon'$ becomes $= \mu$, *while it grows*, the *electric* force grows simultaneously up to $+\infty$, jumps then abruptly over from $+\infty$ to $-\infty$, and grows with $[8\pi/3c^2]\cdot R\varepsilon\varepsilon'$, which became $> \mu$, again continuously from $-\infty$ to 0. If on the other hand $[8\pi/3c^2]\cdot R\varepsilon\varepsilon'$ becomes $= \mu$, *while it decreases*, then simultaneously the *electric* force decays continuously down to $-\infty$, then jumps abruptly over from $-\infty$ to $+\infty$, and decreases, after $[8\pi/3c^2]\cdot R\varepsilon\varepsilon'$ became $< \mu$, again continuously from $+\infty$ to 0.

Both the growth of such a force to infinity, as also the change of its direction, in the instant, at which it became infinite, could now appear as a violation of the continuity found in nature and might be considered as an objection to the general validity of that law, from which such violations of continuity are derived. However, it can be easily proved that these conclusions can not be justifiably drawn at all from that law, namely because these conclusions are linked to completely unrealizable conditions, as was already remarked in Poggendorff's *Annalen*, Vol. 156, p. 29,[67,68,69] which seemingly requires a proof, which finally shall be given here.

[67] [Note by WEW:] It is said at the mentioned location:

> Such a jump of the electric force in *magnitude* and *direction*, namely from $+\infty$ to $-\infty$, really does not occur according to the law, namely because the mass μ with its charge e cannot, in consequence of the always growing acceleration, stay long enough in the interior of the spherical shell, until $[8\pi/3c^2]\cdot R\varepsilon\varepsilon' = \mu$ takes place, but already earlier would have been impelled towards the spherical shell formed by the *rigid* insulator, through whose resistance rest would have been restored, and so the relations presupposed in the calculation would no longer apply.

[68] [Note by HW:] Wilhelm Weber's *Werke*, Vol. IV, p. 333.

[69] [Note by AKTA:] [Web75, pp. 333-334 of Weber's *Werke*].

The acceleration from μ through the previously mentioned *electric* and *non-electric* force obtained by Helmholtz from the fundamental law of electric action resulted in the following equation:

$$q\frac{dq}{ds} = \frac{dV/ds}{\mu - \frac{8\pi}{3c^2} \cdot R\varepsilon\varepsilon'} ,$$

in which the acceleration from μ, instead of $q[dq/ds]$, can also be represented by dq/dt or d^2s/dt^2.

The acceleration is infinitely large, when the value of ε' is $= 3c^2\mu/8\pi R\varepsilon$. The determination of this value of ε', which shall be denoted by η, presupposes that the value of ε has been already determined before. It might appear that conversely also η could be previously determined, by making the determination of ε to depend on the knowledge of η. However it is clear that, after the spherical shell have been charged and η determined, no charge could be carried to the interior of the spherical shell, hence also not the charge ε of the particle μ.

Hence if the charge ε of the particle μ in the interior of the spherical shell is given, then the charge of surface of the spherical shell, for which the force becomes infinite, can be calculated in advance, namely for any unit surface, as was already stated above:

$$\eta = \frac{3c^2}{8\pi} \cdot \frac{\mu}{R\varepsilon} ;$$

however the real *production* of this charge would necessarily be connected with a *gradual* growth of the charge from $\varepsilon' = 0$ to $\varepsilon' = \eta$.

Assuming this, denote the time at which $\varepsilon' = \eta$ occurs by $t = 0$, and the time at which $\varepsilon' = 0$ had occurred by $t = -\vartheta$. If one now further sets the growth of the charge ε' proportional to time, namely

$$\varepsilon' = \eta\left(1 + \frac{t}{\vartheta}\right) ,$$

and if one assumes, in order to simplify the analysis, the center of the spherical shell as the initial point of the trajectory s, where the particle μ at time $t = -\vartheta$ (i.e. at the time where $\varepsilon' = 0$) is at rest, hence with $\varepsilon' = 0$ simultaneously one has $s = 0$ and $q = 0$, and if one finally assumes as *constant* the *non-electric* force $dV/ds = a$ exerted on μ, then from the stated equation, namely from

$$\frac{dq}{dt} = \frac{\frac{dV}{ds}}{\mu - \frac{8\pi}{3c^2} \cdot R\varepsilon\varepsilon'} ,$$

it results after the substitutions $\varepsilon' = \eta(1 + [t/\vartheta])$, $\mu = [8\pi/3c^2] \cdot Re\eta$, and $dV/ds = a$, the equation

$$dq = -\frac{a\vartheta}{\mu} \cdot \frac{dt}{t} \ .$$

The integral of this equation can be written as:

$$q = -\frac{a\vartheta}{2\mu} \cdot \log c^2 t^2 \ .$$

From here it follows, since $q = 0$ for $t = -\vartheta$, that $c^2 = 1/\vartheta^2$.

If one substitutes this value for c^2 in the previous equation and puts ds/dt for q, then one obtains

$$ds = -\frac{a\vartheta}{2\mu} \cdot \log \frac{t^2}{\vartheta^2} \cdot dt \ .$$

From here it follows by integration

$$s = \frac{a\vartheta t}{\mu} \left(1 - \tfrac{1}{2} \log \frac{t^2}{\vartheta^2}\right) + C \ .$$

Since now $s = 0$ for $t = -\vartheta$, it then results $C = a\vartheta^2/\mu$; consequently

$$s = \frac{a\vartheta^2}{\mu} \left[1 + \frac{t}{\vartheta}\left(1 - \tfrac{1}{2} \log \frac{t^2}{\vartheta^2}\right)\right] \ .$$

Both obtained formulas, which, when one denotes the *non-electric* force acting on μ by $a = g\mu$, can be written as:

$$\begin{aligned} q &= -\frac{g\vartheta}{2} \cdot \log \frac{t^2}{\vartheta^2} \ , \\ s &= g\vartheta^2 \left[1 + \frac{t}{\vartheta}\left(1 - \tfrac{1}{2} \log \frac{t^2}{\vartheta^2}\right)\right] \ , \end{aligned}$$

they can now be represented easily in a tabular overview as follows, where e is the base of the natural logarithm:

$\frac{t}{\vartheta}$	$\frac{s}{g\vartheta^2}$	$\frac{q}{g\vartheta}$	$\frac{\varepsilon'}{\eta} = \left(1 + \frac{t}{\vartheta}\right)$
1	0	0	0
$-e^{-1}$	$1 - 2e^{-1}$	1	$1 - e^{-1}$
$-e^{-2}$	$1 - 3e^{-2}$	2	$1 - e^{-2}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$
0	1	∞	1
$\vdots$	$\vdots$	$\vdots$	$\vdots$
$+e^{-2}$	$1 + 3e^{-2}$	2	$1 + e^{-2}$
$+e^{-1}$	$1 + 2e^{-1}$	1	$1 + e^{-1}$
$+1$	2	0	2
$+e$	1	-1	$1 + e$
$+e^2$	$1 - e^2$	-2	$1 + e^2$

One sees from this overview, that the particle μ, which would have been traveled in time ϑ the distance $\frac{1}{2}g\vartheta^2$ under the influence of the acceleration g originating from the *non-electric* force, doubles its distance in the presence of the electric force, and, while without the electric force it would have reached the velocity $g\vartheta$, with the electric force it reaches infinite velocity.

However, with this obtained infinitely large velocity it does not cover the smallest finite distance element, as a consequence of the fact that the then infinite *positive* acceleration abruptly turns into infinite *negative* acceleration, and that as a consequence of this the velocities are equal to one another for the same time period *before* and *after* this instant, according to which the velocity q at time $t = +\vartheta$ (i.e. after it has passed the time period 2ϑ counted from the start of the motion) is equal to the one at the start at time $t = -\vartheta$, namely $q = 0$, where the distance s, when the spherical shell is sufficiently large so that s fits in, would have increased again by $g\vartheta^2$, hence reaching $s = 2g\vartheta^2$. The charge ε' would in the process have reached 2η. But from now on, with continued growth of time and charge, the distance s of the particle μ from the center of the spherical shell would quickly decrease again until $s = 0$, and then become negative until $s = -R$, where the particle μ would hit the spherical shell, at time t, which can be determined from the equation $-R = g\vartheta^2[1 + [t/\vartheta](1 - \frac{1}{2}\log[t^2/\vartheta^2])]$, and with velocity q which, after having determined t, is found to be $= [g\vartheta/2]\log[t^2/\vartheta^2]$.

So far, as already remarked, it was assumed that the radius R of the spherical shell is larger than the maximal value $2g\vartheta^2$, which s reaches at time $t = +\vartheta$. Would R be smaller, then it is automatically clear, that the particle μ would hit the spherical shell sooner, namely in that instant when s had become $= R$, at that time t which could be determined from the equation $R = g\vartheta^2[1 + [t/\vartheta](1 - \frac{1}{2}\log[t^2/\vartheta^2])]$.

But now it shall not be considered a continuing growth of the electric charge, but the case, where the charge, after superseding the value η a bit, remains *constant*. E.g. suppose the constant charge is given by $\varepsilon' = \eta(1 + [1/e^2])$, where μ has the velocity $q = 2g\vartheta$ and is located at distance $s = (1 + [3/e^2])g\vartheta^2$ from the center of the spherical shell.

If one inserts in the equation of Helmholtz

$$\frac{dq}{dt} = \frac{\frac{dV}{ds}}{\mu - \frac{8\pi}{3c^2} \cdot R\varepsilon\varepsilon'}$$

the value $\eta(1 + [1/e^2])$ for ε', so

$$\frac{dq}{dt} = \frac{\frac{dV}{ds}}{\mu - \frac{8\pi}{3c^2} \cdot R\varepsilon\eta\left(1 + \frac{1}{e^2}\right)} \ .$$

Now set herein as earlier $dV/ds = a$ and $[8\pi/3c^2] \cdot R\varepsilon\eta = \mu$, to obtain the differential equation

$$dq = -\frac{ae^2}{\mu} \cdot dt \ ,$$

and through its integration:

$$q = -\frac{ae^2}{\mu} t + C \ .$$

If now the time is counted from that instant at which $\varepsilon' = \eta(1 + [1/e^2])$, then for $t = 0$ the value of $q = 2g\vartheta$ has been found already above, consequently, as set above $a = g\mu$,

$$C = 2g\vartheta \ ,$$

hence

$$q = 2g\vartheta - e^2 gt \ ,$$

or

$$ds = \left(2g\vartheta - e^2 gt\right) dt \ ,$$

from where by integration

$$s = 2g\vartheta t - \frac{e^2}{2} gt^2 + C' \ .$$

Now it was found, if one counts the time from that instant at which $\varepsilon' = \eta(1 + [1/e^2])$, for $t = 0$ the value of $s = (1 + [3/e^2])g\vartheta^2$, consequently

$$C' = \left(1 + \frac{3}{e^2}\right) g\vartheta^2 \ ,$$

hence

$$s = \left(1 + \frac{3}{e^2}\right) g\vartheta^2 + 2g\vartheta t - \frac{e^2}{2} g \cdot t^2 \ .$$

This formula together with the preceding one

$$q = 2g\vartheta - e^2 gt \ ,$$

can now be easily and clearly arranged, as the previous formulas for s and q, in the form of the following Table:

$\frac{t}{\vartheta}$	$\frac{s}{g\vartheta^2}$	$\frac{q}{g\vartheta}$	$\frac{\varepsilon'}{\eta}$
0	$1 + \frac{3}{e^2}$	2	$1 + \frac{1}{e^2}$
1	$3 + \frac{3}{e^2} - \frac{e^2}{2}$	$2 - e^2$	$1 + \frac{1}{e^2}$
2	$5 + \frac{3}{e^2} - 2e^2$	$2 - 2e^2$	$1 + \frac{1}{e^2}$

This Table can be easily continued further; but one already sees from here that from $t = 2\vartheta/e^2$ on, after the charge became constant, the distance s of the particle μ from the center of the spherical shell decreases and becomes very soon negative, until eventually the particle μ, when $s = -R$ has been reached, hits the spherical shell, at time t and with the velocity q, which both can be determined from the two equations

$$\begin{aligned} -R &= \left(1 + \frac{3}{e^2}\right) g\vartheta^2 + 2g\vartheta t - \frac{e^2}{2} gt^2 \ , \\ q &= 2g\vartheta - e^2 gt \ . \end{aligned}$$

One sees from this presentation of the whole process in its *context*, that none of the “inconsistent and absurd” consequences, with which Helmholtz wanted to disprove the established fundamental law, really occurs.

Helmholtz’s objection (A) in Borchardt’s *Journal*, Vol. 72, p. 61 and Vol. 75, p. 38 has not yet been discussed.[70] It consists in the claim, that the established fundamental law of electric interaction, or rather the differential equations of Kirchhoff originating from this law,[71] would lead to an unstable

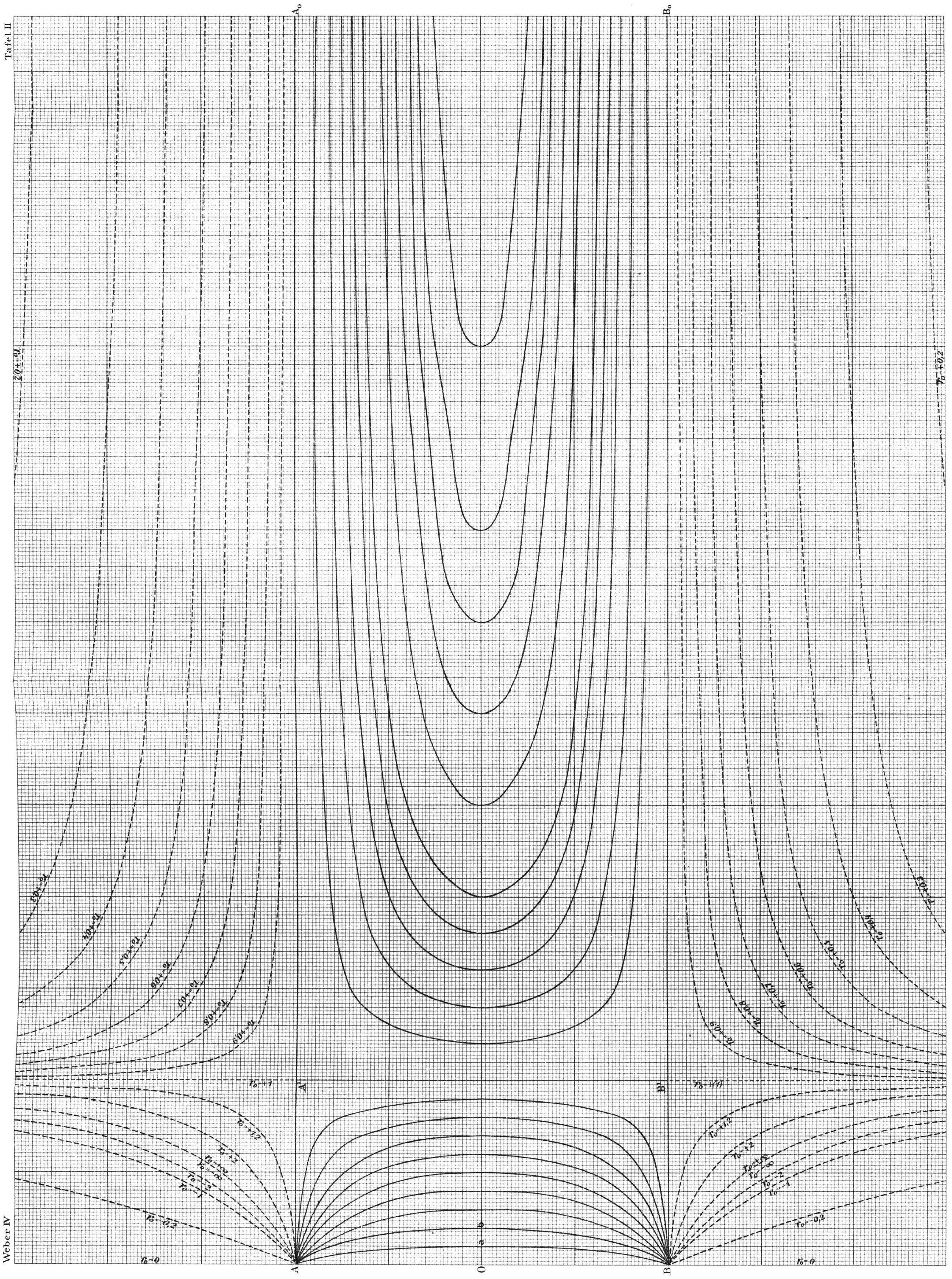
[70] [Note by AKTA:] [Hel70, p. 61] and [Hel73, p. 38].

[71] [Note by AKTA:] [Kir57] with English translation in [GA94].

equilibrium of the electric matter, or rather to a motion of this matter, whose velocity would grow to infinity in time. But Neumann has already proved in the *Berichte der Königl. Sächs. Gesellschaft*, October 1871, p. 477,[72] that the differential equations of Kirchhoff rest, apart from that fundamental law, on yet various other accessory assumptions, and that consequently that law cannot be cast in doubt due to general concerns raised against these differential equations.

After this correction already given by Neumann, for which it is to be referred the closer discussion given by Neumann in his Memoir, pp 128-149 of the present Volume,[73] a further discussion of this objection is not required. Such discussion would lie, since it mainly would yet concern only those accessory assumptions, completely outside of the limits that are imposed on the present Memoir.

[72] [Note by AKTA:] [Neu71, pp. 477-478].
[73] [Note by AKTA:] [Neu74, pp. 128-149].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